Modelling and Nonlinear Control of a Low-power Gas Turbine
(Theses)

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1 Motivation and aim of the research work

Recently, there has been a growing need for control systems of gas turbine engines because of the importance of their application areas (aviation, electric power trade, gas transport and military application) and because of the demands on their operation and the safety requirements of them. The scope of their duties has been widened and the requirements on their accuracy (tracking of the characteristics with maximum efficiency) has also been increased. Fortunately, at the same time, the area of control design methodology has significantly improved, and nonlinear techniques are gaining more and more ground [6] in addition to the traditional linear methods. This development has special importance in case of process- (for example thermodynamical) systems [5] as well as in the case of electrical and mechanical systems.

From the point of view of dynamic modelling, gas turbines can be regarded as a mixed thermodynamical-mechanical systems. The dynamic conservation balance and the algebraic equations of the model are nonlinear, so they represent a complex problem both form system analysis and control point of view. Therefore, the most common way to perform dynamic analysis and control design is to apply a locally linearized model around an operating point instead of considering a nonlinear model. On the other hand, new mathematical and control methods and tools has recently came to the front of the research and were developed which are able to handle nonlinear systems and models using special nonlinear techniques. Thus the results of the dynamic analysis of nonlinear models can be properly applied to nonlinear control design [6], [10].

Based on these facts the aim of the research work summarized in my dissertation was to investigate the possibility of designing a nonlinear controller satisfying prescribed control specification for a gas turbine with simple structure and working cycle (Chapter 2) [1] and to design and tune a possible nonlinear controller.

In order to reach the above described research aim, the dynamic model of the investigated gas turbine suitable for control purposes has to be developed [9], which is the subject of the first part of my dissertation. The model can be set up from thermodynamical and mechanical principles including suitable constitutive relations (Chapter 3) [5]. An important step of modelling is the transformation of the differential equations with the algebraic equations substituted into a nonlinear state space model and identifying the ingredients of this model written standard nonlinear input-affine form. The developed dynamic mathematical model has unknown static and dynamic parameters.

Having estimated these parameters [7], the verification of the dynamic model is performed by open-loop simulations (Chapter 4). An important step of prescribing the control aims is the investigation of the sensitivity of the developed model with respect to its signals and determining the operating region of the gas turbine and consequently that of the developed model.

The subject of the second part of my dissertation is the dynamic analysis of the gas turbine and its controller design using the developed and verified model. The first and basic step of controller design is the dynamic analysis of the locally linearized and the nonlinear model (Chapter 5) [6], [11]. The determination of the local dynamic properties of the linearized model is a relatively simple task. In the case of a nonlinear model, however, the dynamic analysis requires complicated computations and it is often not feasible.

Based on the results of dynamic analysis and the specification of the control aim, a linear [2], [4] and a nonlinear controller [3] have been selected from the possible control structures and their design and tuning is presented in Chapter 6 and Chapter 7. The linear controller serves as a reference case. Its performance, qualitative and quantitative properties is compared with that of the nonlinear controller using computer simulations in Chapter 8.

2 Methods and tools

A Deutz T216 gas turbine engine used for generating power is installed in the Budapest University of Technology and Economics, Department of Aircraft and Ships on a test-stand [1]. The data acquisition system of the test-stand makes the measurements of static operating points and dynamic, transient processes possible. In both the static and dynamic cases, the total pressures and temperatures of the typical points of the gas turbine cycle, the number of revolutions, the moment of load and consumption of fuel are the measurable quantities. This gas turbine is the subject of modelling and control design studies presented in this thesis.

Modelling is carried out by executing a systematic modelling procedure that consists of a fixed sequence of steps [5]. First of all the object and the aim of modelling has to be defined, which highly influence the final form of the model. For dynamic models dynamic (differential) equations are needed, which can be obtained from conservation balances; while the algebraic equations are further static relations. The modelling assumptions have to be taken into consideration in a consistent way throughout the whole model development procedure. In order to obtain a state-space model in its standard form, the possibility of substituting the algebraic equations into the differential equations has to be investigated. If all algebraic equations can be substituted into the differential equations in explicit manner, then the final result of the modelling is a set of differential equations, which can be transformed relatively easily into a nonlinear state-space model in its input-affine form.

The dynamic models generally contain unknown static and/or dynamic parameters [7]. If a model is linear in its static parameters, then a standard least squares method is applicable for parameter estimation. However, when the dynamic model with unknown dynamic parameters is nonlinear in its parameters, then an optimization problem has to be solved using a suitable method, for example the Nelder-Mead simplex search algorithm [8]. In all cases the accuracy, the "goodness" of the estimation and the variance of the estimated parameters have to be investigated.

The next important step of modelling is the determination of the validity and the operating region of the model. The modelled system has to be investigated only within this region, its dynamic properties have to be established and the control has to be designed. Finally, the validation of the dynamic model has to be performed. It means, that the model has to be checked using our engineering knowledge and intuitions, knowing the real system. The validation of a model with very complicated algebraic structure can be performed using computer simulations in a most suitable manner.

The dynamic analysis of the prepared model has primary importance in control design. Dynamic analysis means checking three basic dynamic properties of the system: as controllability, observability and stability. In case of a linear model controllability and observability test can be performed using the investigation of ranks of Kalman controllability and observability matrixes, while the stability can be tested by investigating the eigenvalues of the state matrix of linearized model [11]. In case of nonlinear input-affine state-space models the dynamic properties can be very different in different regions of the state space [6], [10]. The investigation of them is possible using algorithms based on the extension of the algorithms applicable in the linear case. But these methods need complicated computations, so they are often not feasible. In these cases the existence of the dynamic properties can be proved using different considerations.

The results of the dynamic analysis highly influence the possibilities, properties and problems of control design. First of all, the control aims have to be set for any control design, which contain the requirements of control in details. Having specified the control aims, we can select from the possible linear and nonlinear control structures those types, which satisfy the control aims and fit to the results of the dynamic analysis. For example, a control Lyapunov-function based nonlinear controller can guarantee the globally asymptotic stability of the closed-loop system, which property is not proved by dynamic analysis of the nonlinear model in this case [3].

Having designed the controllers and performing their tuning suitable for the prescribed quantitative control aim, an important problem is the comparison and evaluation of the controllers. For this purpose, qualitative and quantitative criteria have to be determined in advance starting from the control aims. As a result of the comparison of controllers we can state, which controller is more suitable for the given control problem, which controller shows better qualitative and quantitative properties.

3 New scientific results

Thesis 1 *Nonlinear dynamic model of the investigated gas turbine* (Chapter 3)
(**[P1]**, **[P2]**, **[P3]**)
The nonlinear dynamic model of the gas turbine considered as a mixed thermodynamical-mechanical system suitable for control purposes has been built and verified using a systematic and checked modelling methodology. It has been shown that the model exhibits the following special properties:

- 1.1 The dynamic model of the gas turbine is given by nonlinear differential-algebraic set of equations. The differential equations are balance equations for the mass and internal energy of the combustion chamber as a balance volume and for conservation of the mechanical energy of the rotating part.
- 1.2 It has been shown that the 3 state equations of the nonlinear dynamic model can be rewritten into standard input-affine form.
- 1.3 The coordinate-functions of the dimensionless and centered nonlinear model have the following properties:
  - 1.3.1 the coordinate-functions $f(x)$ depend also on the elements of the disturbance-vector: $f(x) = f(x, d)$,
  - 1.3.2 the coordinate-functions $g(x)$ do not depend on the state variables: $g(x) = B = const.$,
  - 1.3.3 the coordinate-functions of the output equations have the following forms: $h_1(x) = h_1(x, d_1)$, $h_2(x) = x_2$ and $h_3(x) = x_3$.

Thesis 2 *Estimation of the unknown parameters of the nonlinear dynamic model* (Chapter 4)
(**[P3]**, **[P4]**)
The dynamic model of the investigated gas turbine has been verified using measured data. By estimating the unknown static parameters of the model it has been shown that the simpler bilinear expressions used for the approximation of the characteristics of the compressor and the turbine are sufficiently accurate. Furthermore,

the dynamic model together with the estimated dynamic parameters is also adequately (approximately 1 %) accurate.

2.1 The static parameters of the model are the unknown coefficients of the polynomials approximating the characteristics of the compressor and the turbine. The parameters have been estimated using static measurements by applying the least squares method.

2.2 The dynamic parameters were estimated using measured step responses and the simulation model that is nonlinear in the unknown parameters. The nonlinear optimization problem required for the parameter-estimation has been solved using the Nelder-Mead simplex search algorithm.

Thesis 3 *Nonlinear dynamic analysis of the nonlinear dynamic model* (Chapter 5)
(**[P3]**)
Nonlinear dynamic analysis has been performed on the nonlinear dynamic model of the gas turbine. The analysis results show that the model:

3.1 is stabilizable, since it is possible to design a control Lyapunov-function based stabilizing nonlinear feedback law for the system. Moreover, an arbitrarily small neighborhood of any operating point within the operating region is reachable from any initial state with this feedback.

3.2 is observable in the nonlinear sense.

Thesis 4 *Linear and nonlinear control structure selection and controller design* (Chapter 6, 7, 8)
(**[P5]**)
Starting from the exact engineering problem statement of the control aims the LQ servo structure has been selected as a linear reference case and a control Lyapunov-function based block-structured nonlinear controller has been designed. As a result of the comparison based on computer simulation experiments it has been shown, that the system controlled by the control Lyapunov-function based nonlinear controller exhibits similar or more advantageous properties than the system controlled by the linear LQ servo controller.

4.1 An LQ servo controller satisfying the predefined control aims has been designed and tuned.

4.2 A control Lyapunov-function based block-structured nonlinear controller has been designed and tuned for the nonlinear dynamic model of the investigated gas turbine. The properties of the closed-loop system and the controller blocks responsible for them are the following.

  4.2.1 Any operating point within the operating region can be set and it is globally asymptotically stable (block guaranteeing stability, block guaranteeing asymptotic stability and block guaranteeing asymptotic stability of the further operating points).

  4.2.2 The number of revolutions follows the reference signal defined by the throttle (block guaranteeing asymptotic stability of the further operating points).

  4.2.3 The protection of the gas turbine against the extreme number of revolutions and temperature is guaranteed in any operating point (block guaranteeing the protection of the gas turbine).

  4.2.4 The response of the number of revolutions is insensitive to the change of the elements of the disturbance vector (block guaranteeing asymptotic stability of the further operating points).

  4.2.5 The closed-loop system is robust with respect to the time-varying, unmeasurable parameters (block guaranteeing robustness).

  4.2.6 By proper tuning of the nonlinear controller parameters the closed-loop system has the prescribed settling time.

4 Publications related to the thesis

[P1] **P. Ailer**, I. Sánta, G. Szederkényi and K. M. Hangos. Nonlinear Model-Building of a Low-Power Gas Turbine. *Periodica Polytechnica Ser. Transportation Engineering* 2001. **29**/1-2. pp. 117-135. (**Thesis 1**)

[P2] **P. Ailer**. Nonlinear Mathematical Modelling and Control Design Developed for Gas Turbine. *Proceedings of the 7th Mini Conference on Vehicle System Dynamics, Identification and Anomalies* Budapest, editor: István Zobory, published by the Technical University of Budapest, 2000. pp. 465-472. (**Thesis 1**)

[P3] **P. Ailer**, G. Szederkényi and K. M. Hangos. Modelling and Nonlinear Analysis of a Low-Power Gas Turbine. *Research Report of the Systems and Control Laboratory SCL-1-2001* Budapest, Computer and Automation Research Institute HAS, 2001. 25 p. (**Thesis 1, Thesis 2, Thesis 3**)

[P4] **P. Ailer**, G. Szederkényi and K. M. Hangos. Parameter-Estimation and Model Validation of a Low-Power Gas Turbine. *Proceedings of the "Modelling, Identification and Control'2002" Conference* Innsbruck, Austria, editor: M. H. Hamza, published by the ACTA Press, 2002. pp. 604-609. (**Thesis 2**)

[P5] **P. Ailer**, G. Szederkényi and K. M. Hangos. Model-Based Nonlinear Control of a Low-Power Gas Turbine. *Proceedings of the "15th IFAC World Congress on Automatic Control"* Barcelona, Spain, editors: E. F. Camacho, L. Basanez, J. A. de la Puente, published by the Elsevier Science, 2002. paper no.: 755. 6 p. (**Thesis 4**)

5 Publications partially related to the thesis

[O1] P. Ailer. Az RD-33-as Hajtómű Centrifugális Fordulatszám-Szabályozójának Matematikai Modellezése. *ZMNE Repüléstudományi Közlemények* 1998. **X. 24.** pp. 175-191.

[O2] P. Ailer and I. Sánta. Mathematical Modelling and Dynamic Analysis of Rotational Speed Control System of Low Bypass Ratio Turbofan. *Proceedings of the* 6^{th} *Mini Conference on Vehicle System Dynamics, Identification and Anomalies* Budapest, editor: István Zobory, published by the Technical University of Budapest, 1998. pp. 337-348.

[O3] P. Ailer. Kis Teljesítményű Gázturbina Szabályozásának Matematikai Modellezése. *ZMNE Repüléstudományi Közlemények* 1999. **XI. 26.** pp. 239-250.

[O4] P. Ailer. Mathematical Modelling of Control System of Low-Power Engine. *Proceedings of "The Challenge of Next Millenium on Hungarian Aeronautical Sciences" Conference* Budapest-Nyíregyháza, editors: József Rohács, Gyula Szabó, Piroska Ailer and Árpád Veress, published by the eR-GROUP, 1999. pp. 142-152.

[O5] P. Ailer. Gázturbina-Egységek Karakterisztikája, a Gázturbina Matematikai Modellje. *ZMNE Repüléstudományi Közlemények* - 2000. **XII. 29.** pp. 139-147.

[O6] P. Ailer. Comparison of Linear and Non-linear Mathematical Models Developed for Gas Turbine Control. *Proceedings of the* 3^{rd} *International Conference on Nonlinear Problems in Aviation and Aerospace* Daytona Beach, Florida, USA, editor: Seenith Sivasundaram, published by the European Conference Publications, Cambridge, UK, 2000. pp. 11-19.

[O7] P. Ailer. Mathematical Modelling of a Low-Power Gasturbine Engine and Its Control System. *Proceedings of the* 22^{nd} *International Congress of Aeronautical Sciences* Harrogate, Great Britain, editor: J. P. Marec, published by the Optimage Ltd., 2000. paper no.: 752. 7 p.

[O8] P. Ailer. Mathematical Modelling of Gas Turbine Engine and Its Control System. *Periodica Polytechnica Ser. Transportation Engineering* (in print)

6 Application of the results

The modelling, static and dynamic parameter-estimations and control methods used for the investigated gas turbine and shown in my dissertation can be applied to other gas turbines with different types and constructions with more complicated structures (for example a twin-spool engine with low/high bypass ratio and/or with afterburning) in a relatively straightforward way.

In case of other construction, however, the modelling equations are changed, the number of them is generally increasing. For example, in the case of a twin-spool engine two dynamic equations for conservation of the mechanical energy are needed in the model. Because of the twin-spool there are two (one low pressure and one high pressure) compressors and two turbines, so the number of the algebraic equations approximating the characteristics are doubled.

There are more inputs in the general case, for example in the case of afterburning the mass flow rate of fuel fed into the afterburner is an additional input variable. If some parts of the gas turbine have variable geometry, then these variable geometries (for example of the compressor or the nozzle) are also the elements of the input-vector.

The number of variables appearing in the control aim is also increasing in a more realistic case. Not only the number of revolutions but the total temperature of the afterburner has to be held constant if the afterburner works.

Despite of the differences above, the investigated gas turbine can be regarded as an important basic case, which serves as a model to solve control problems for a gas turbine with more complicated structures, since the sequence of problems to be solved and the applicable methods in both cases essentially correspond.

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