Cascade stabilization
of a
simple nonlinear limb model

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The proof of stability of a cascade control using feedback linearization and pole placement in the case of an elbow-like nonlinear limb model, similar to the one described in [2], is proposed in this article. The proof is based on the backstepping technique described by van der Schaft in [8]. In the case of regulation the feedback properties are examined. The controller structure is extended to the task of trajectory following. Simulations are performed to test the theoretical basis for control design - in the trajectory following case, a sinusoid trajectory should be followed.

1 Introduction

Application of nonlinear control for biomechanical systems has appeared in the literature several times in the past decades [6], [1], but the proof of stability for such nonlinear control methods is not prevalent in the literature. The proof of stability in the case of nonlinear system and control theory is always a challenge for systems with complex state-space models. In this case the system described has a quite complex dynamics, but we can avoid some of these difficulties by feedback linearization, and by the backstepping method.

Motivation and Aim:

Even the simplest limb model exhibits strongly nonlinear dynamic behavior that calls for the application of the results of nonlinear systems and control theory. The control of musculoskeletal structures has considerable importance in the field of human locomotion control, designing and controlling muscle prothesis and artificial limbs. Furthermore the techniques of FES (functional electrical stimulation) - of patients with some kind of paralysis - can be improved with appropriate control methods.

The aim of this study is to prove the stability of a nonlinear cascade control (via feedback linearization [4] or [5]) for a simple limb model.

2 The simple nonlinear limb model

Material and Methods:

A nonlinear input-affine state-space model has been developed for a simple one-joint system with a flexor and an extensor muscle (see figure 1) which is suitable for nonlinear system analysis and control. The model takes the nonlinear properties of the force-length relation and the force-contraction velocity relation into account.

Exerted forces depend linearly on the activation state of muscles, following the principles in [10], [9] and [7]. In this case we use a simplifed version of the model described in [2], and do not take the tendon dynamics into account, as described in [3]. The inputs of the model are the normalized activation signal of muscles, the output is the joint angle, and the number of state variables is 4.

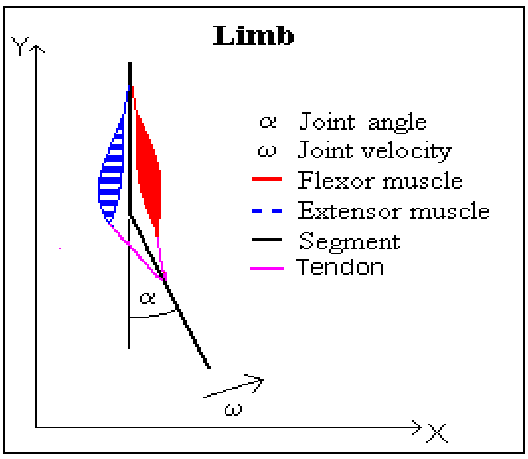

Figure 1: The system

As preliminary model analysis we performed stability, controllability and observability analysis of the linearized model around steady-state points. The result showed, that the stability of the model strongly depends on the steady-state point. In this study we will examine a steady-state point, which exhibits instable properties in open loop case. Utilizing the cascade structure of the system (the muscle dynamics do not depend on the limb dynamics), a cascade control (feedback linearization and pole-placement for the limb dynamics and expanding the control to the muscle dynamics with backstepping) was designed.

2.1 Structure and signal flow of the model

As it can be seen in figure 2, the dynamics of the muscle activation do not depend on the dynamics of the limb. So, the muscle activation states can be defined as inputs to the dynamics of the segments.

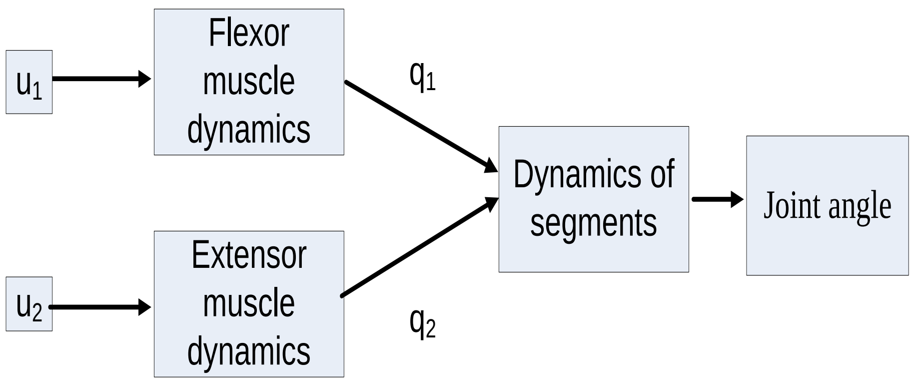

Figure 2: The cascade structure of the system

2.2 Equations of the model

2.2.1 Segmental dynamics

The dynamics of the segments in open-loop case can be described with the following equations:

$$\begin{aligned} \frac{\partial\alpha}{\partial t} &= \omega \\ \frac{\partial\omega}{\partial t} &= \frac{1}{\Theta+ml_{COM}^2}(M_m(q_1,q_2,\alpha,\omega)+ml_{COM}cos(\alpha+\xi)g) \end{aligned} \tag{1}$$

where α [rad] is the joint angle, ξ [rad] is the angle between the global coordinate-system's x axis, and the not-moving upper segment of the limb (in our model ξ is always equal to $-\pi/2$), ω [rad/s] is the angle velocity, Θ $[kgm^2]$ is the moment of inertia defined to the mass-centre point of the bone, m [kg] is the mass of the moving limb part, l_{COM} [m] is the distance between the moving limb part's center of mass point and the joint axis, M_m [Nm] is the resulting joint torque of the muscles, and g $[m/s^2]$ is the vector of gravitational acceleration. q_1 and q_2 denotes the activation states of the muscles.

If we study the model around $\alpha = \pi/2$ joint angle, the forces of ligaments, bones and the passive force of the muscles can be neglected, so we can write

$$M_m = F_{max}^f f_{LM}^f(\alpha) f_{VM}^f(\alpha,\omega) d^f q_1 + F_{max}^e f_{LM}^e(\alpha) f_{VM}^e(\alpha,\omega) d^e q_2 \tag{2}$$

where f_{LM}^f and f_{LM}^e denote the function of the force-length characteristics in the case of the flexor and extensor muscle, f_{VM}^f and f_{VM}^e denote the function of the force-contraction velocity characteristics in the case of the flexor and extensor muscle. These characteristics are fully described in [3], and they can be seen in the next figures:

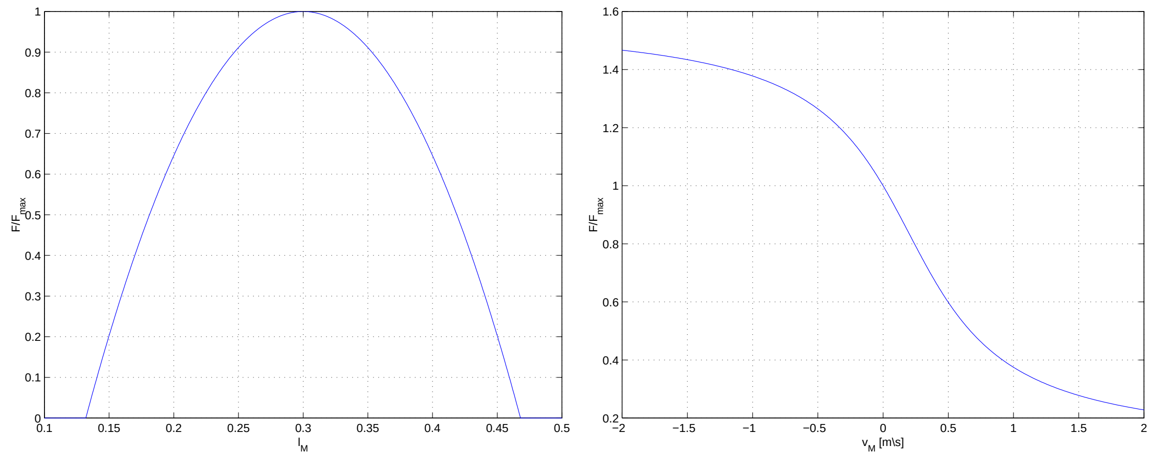

Figure 3: F_{LM} and F_V

d^f and d^e [m] denote the moment arms for the flexor and extensor muscle, the distance between the axis of the joint and the point where the forces appear, and F^f_{max} and F^e_{max} denote the maximal force of the flexor/extensor muscles.

2.2.2 Muscle dynamics

The differential equation of the muscle defines the connection between $q(t)$, the activation state of the muscle and the activation signal $u(t)$. With $u(t) \in [0, 1]$ the equation taken from Zajac [10] is:

$$\frac{dq}{dt} = -\left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u(t)\right) q + \frac{1}{\tau_{act}}u(t) \tag{3}$$

where τ_{act} [s] is the activation time, showing how quick the muscle reacts on the external activation signal coming from the nerval system. β is a constant, describing the correlation between the decrease of the activation state and the external activation signal. If $\beta = 1$ then the external activation signal does not affect the decrease of the activation state, if $\beta = 0$ then it strongly affects the decrease. $q_1(t)$ denotes the activation state of the flexor muscle, and $q_2(t)$ denotes the activation state of the extensor muscle.

2.2.3 State-space equations

With the notation x_i for the state-space variables ($x_1 = q_1$, $x_2 = q_2$, $x_3 = \alpha$, $x_4 = \omega$), the equations are as follows:

$$\begin{aligned}
\tfrac{dx_1}{dt} &= -\left(\tfrac{1}{\tau_{act}}(\beta + [1-\beta]u^f(t))\right) x_1 + \tfrac{1}{\tau_{act}}u_1(t) \\
\tfrac{dx_2}{dt} &= -\left(\tfrac{1}{\tau_{act}}(\beta + [1-\beta]u^e(t))\right) x_2 + \tfrac{1}{\tau_{act}}u_2(t) \\
\tfrac{dx_3}{dt} &= x_4 \\
\tfrac{dx_4}{dt} &= \tfrac{1}{\Theta + ml^2_{COM}}(M_m(x_1, x_2, x_3, x_4) + ml_{COM}cos(x_3 + \xi)g_y)
\end{aligned} \tag{4}$$

where the first two equations describe the dynamics of the muscles, and the second two describe the dynamics of the limb. $u_1(t)$ denotes the activation signal of the flexor muscle, and $u_2(t)$ denotes the activation signal of the extensor muscle.

If we rearrange equation 4 we can get the following general form of input-affine systems:

$$\begin{aligned}
\dot{x} &= f(x) + \textstyle\sum_{i=1}^{m} g_i(x)u_i(t) \\
y &= h(x)
\end{aligned} \tag{5}$$

where

$$f(x) = \begin{pmatrix} -\frac{1}{\tau_{act}}\beta x_1 \\ -\frac{1}{\tau_{act}}\beta x_2 \\ x_4 \\ \frac{1}{\Theta + ml_{com}^2}(M(x_1, x_2, x_3, x_4) + ml_{com}cos(x_3)g_y \end{pmatrix}$$

$$g_1(x) = \begin{pmatrix} -\frac{1}{\tau_{act}}(1-\beta)x_1 + \frac{1}{\tau_{act}} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad g_2(x) = \begin{pmatrix} 0 \\ -\frac{1}{\tau_{act}}(1-\beta)x_2 + \frac{1}{\tau_{act}} \\ 0 \\ 0 \end{pmatrix}$$

2.3 The cascade structure of the model

As we can also see in figure 2, the dynamics of the limb model has a cascade structure. The complete dynamics of the limb can be divided into two parts:

- The activation dynamics of the muscles: This means two simple first order system, with the activation signals as input and the activation states as output.
- The movement dynamics of the limb with the activation states as input and the joint angle as output.

3 Regulation

3.1 Control structure design

We can utilize the cascade structure of the model, and design a controller for the limb dynamics, and from it we derive an another one for the muscle. In this case the actuation signal of the limb dynamics is defined for the muscle as reference signal.

3.2 Control of segments

In the case of the segments, we apply feedback linearization, and pole-placement. In this case we use only the flexor muscle, as input to get a SISO structure. We apply the control to the normalized variables around a steady-state point at $\alpha = \pi/2$ joint angle. The coordinates of the steady-state point are the following:

$$
\begin{aligned}
x_1 &= 0.14558374176266 \\
x_2 &= 0.1 \\
x_3 &= \pi/2 \\
x_4 &= 0
\end{aligned}
$$

As described in [4] for a nonlinear n-dimensional SISO system we need to apply the feedback

$$u = \frac{1}{L_g L_f^{n-1} h(x)} (-L_f^n h(x) + v(t)) \tag{6}$$

and a suitable nonlinear coordinate transformation to obtain: a linear system of order n which is influenced by the input u - including the external input $v(t)$. $L_f h(x)$ denotes the Lie-derivative of $h(x)$ along f, $L_g h(x)$ denotes the Lie-derivative of $h(x)$ along g.

This means, that the state-space model of the feedback linearized closed loop system is the following in the new coordinates:

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= v \\
y &= z_1
\end{aligned} \tag{7}
$$

where $L_f h(x)$ denotes the Lie-derivative of $h(x)$ along f. z_1 and z_2 can be determined by using the coordinate-transformation $z_i = L_f^{i-1} h(x)$.

The state matrices of the new linear system have the following form:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

For the system in the new co-ordinates we can design a pole-placement control:

$$v = -Kz \tag{8}$$

where K is a suitable state-feedback vector. In this case the closed loop system has the Ljapunov function $V(z) = z_1^2 + z_2^2$ in the new co-ordinates.

In fact, we can not act on the system at the point of muscle activation states, so we have to define the value of equation 6 as reference signal for the flexor muscle.

3.2.1 Backstepping

As detailed in [8] if we have the system structure

$$\begin{aligned} \dot{z} &= f(z) + g(z)\xi \\ \dot{\xi} &= a(z,\xi) + b(z,\xi)u \end{aligned} \tag{9}$$

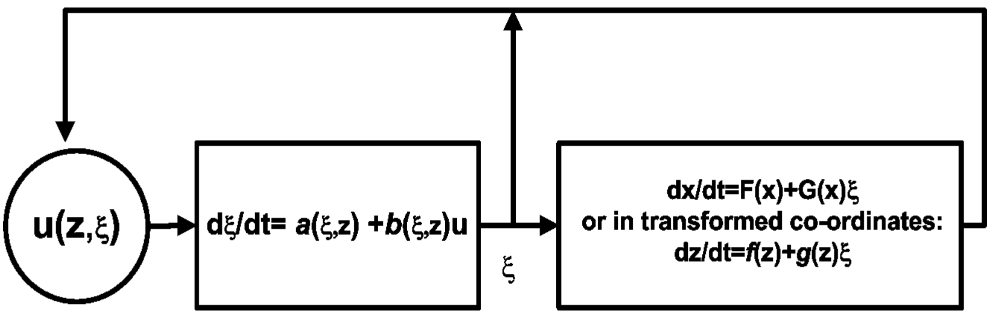

Figure 4: General structure for the backstepping method

and there exists a virtual feedback

$$\xi = \alpha(z) \tag{10}$$

such, that $z = 0$ is an asymptotically stable equilibrium of

$$\dot{z} = f(z) + g(z)\alpha(z) \tag{11}$$

with a Ljapunov function V that is positive definite at $z = 0$, then the system

$$\Sigma : \begin{array}{c} \dot{z} = f(z) + g(z)\xi \\ \dot{\xi} = a(z,\xi) + b(z,\xi)u \end{array} \tag{12}$$

we define $y_1 = \xi - \alpha(z)$, and prescribe the error dynamics $\dot{y_1} = -A_1 y_1$ (where A_1 is a speed influencing constant), then we can write:

$$\dot{y_1} = \dot{\xi} - \dot{\alpha(z)} = a(z,\xi) + b(z,\xi)u - \frac{d\alpha}{dz}\dot{z} = -A_1 y_1 = A_1(\alpha(z) - \xi) \tag{13}$$

we can rearrange the equation to get:

$$u = b^{-1}(z,\xi)(A_1(\alpha(z) - \xi) - a(z,\xi) + \frac{d\alpha}{dz}\dot{z}) \tag{14}$$

leading to the feedback transformed system

$$\begin{aligned} \dot{z} &= [f(z) + g(z)\alpha(z)] + g(z)y_1 \\ \dot{y_1} &= -A_1 y_1 \end{aligned} \tag{15}$$

with storage function $S_1 = V(z) + \frac{1}{2}y_1^2$, satisfying

$$\frac{dS_1}{dt} \leq -\|y_1\|^2. \tag{16}$$

Remark

In our case $\dot{z} = f(z) + g(z)\xi$ means the dynamics of the limb, and $\dot{\xi} = a(z,\xi) + b(z,\xi)u$ denotes the muscle dynamics. Furthermore $a(z,\xi)$ and $b(z,\xi) = a(\xi)$ and $b(\xi)$. From equations 1 and 2 it can be easily seen, that the limb dynamics can also be transformed to the form of 15, because q_1 and q_2, which are considered as inputs, appear linear in the state-space equation. The $\alpha(z)$ virtual feedback means the feedback linearization and the pole placement in this case. Furthermore we have to note that in this case the transformed co-ordinates are identical to the original ones, so $z_1 = \alpha$, $z_2 = \omega$.

3.3 Muscle control

We have to use equation 14 for determining the muscle activation signal.

3.4 Feedback properties

We can analyze the properties of the feedback law. At first we can depict the prescribed reference signal for muscle activation at different joint angle, and joint angle velocity values:

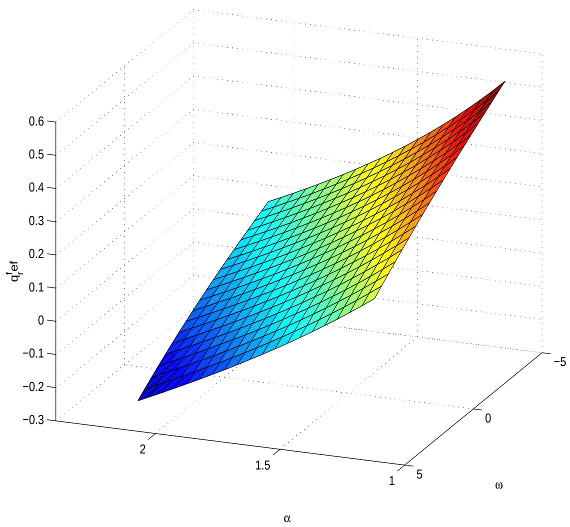

Figure 5: The virtual feedback for q_1

We can see that negative values for the prescribed activation state appear only at points where the error of the joint angle and the value of the joint angle velocity are both quite large positive.

Second, the final activation signal of the muscle depends also on q_1, we can depict for example this function at $\omega = 0$.

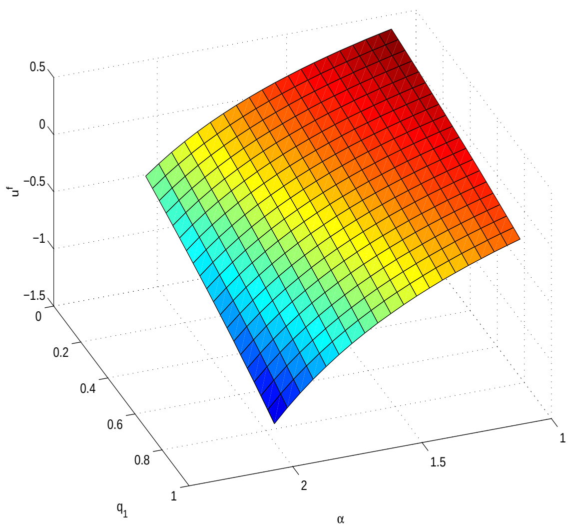

Figure 6: The activation signal

Negative activation signal appears only in the case of big activation states which are unusual in normal functioning.

3.5 Simulation results

Simulations were performed using MATLAB to determine the symbolic expressions needed for the feedback computing, and numerically solving the differential equations. The poles were set to [-15 -20], and A_1 was set to -100. The starting values were the same as the co-ordinates of the steady state point, except the joint angle ($\alpha = x_3$) which was 0.2. In the next figures, the results can be seen:

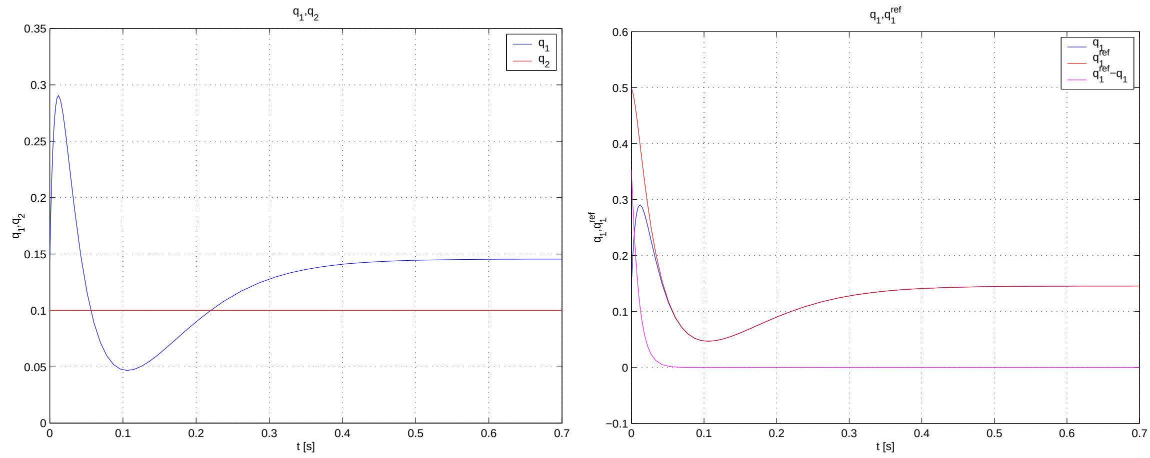

Figure 7: Muscle activation states, the reference signal for muscle activation and the error

As we can see, the error has stable linear dynamics.

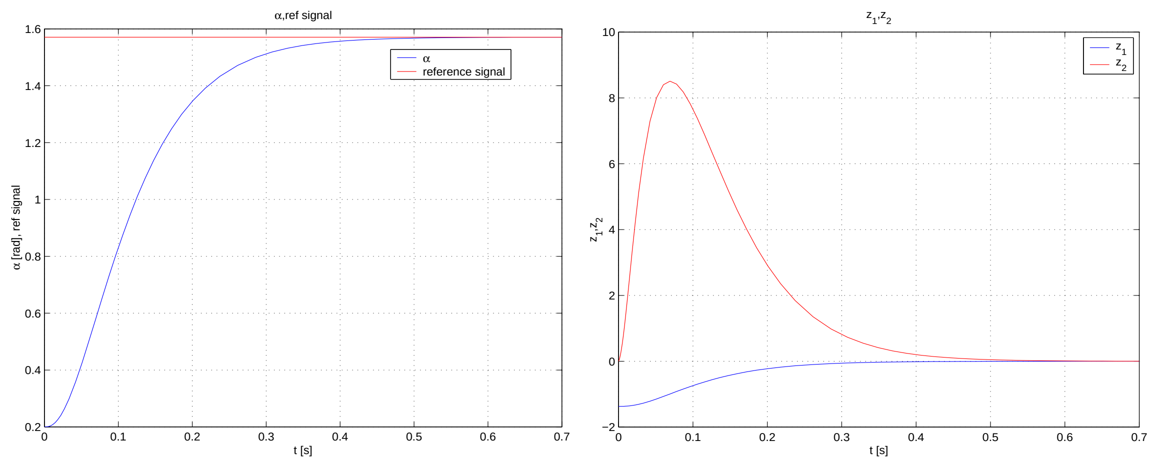

Figure 8: Output: Joint angle, reference signal and the transformed coordinates

In figure 9 we can see, that the input does not brake the input constraint ($u(t) \in [0, 1]$), even in this case, when the initial position is very far from the reference.

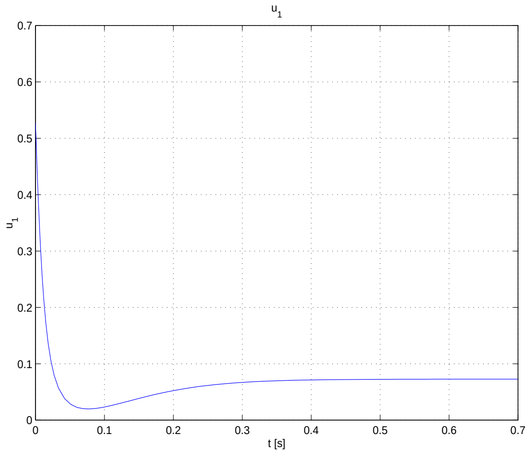

Figure 9: Input: Activation signal of the flexor muscle

4 Trajectory following

In the following we expand the control system further to be able to follow a trajectory. At first, the control aim will be to follow a trajectory of the joint angle velocity. In this case we do not use the centered variables around a steady-state point and we use only the activation signal of the flexor muscle as input. The activation signal of the extensor muscle is constantly 0. We use the same feedback-linearization technique, as in the case of regulation, but now instead of the way of pole-placement in the transformed coordinates we use a different method to determnie the external input v.

As we mentioned above, we have the system structure:

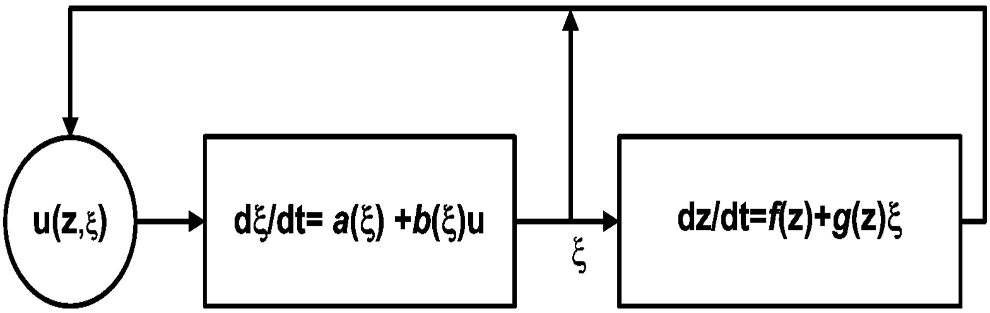

Figure 10: System structure

where

$$f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \qquad g = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \tag{17}$$

$$f_2(z) + g_2(z)\xi_{ref} \doteq v \tag{18}$$

$$\xi_{ref} = \frac{-f_2(z)}{g_2(z)} + \frac{1}{g_2(z)} v \doteq \alpha(z) + \beta(z)v \tag{19}$$

$$\dot{z}_2 = v$$

We define the error of z_2:

$$\overline{z}_2 = z_2 - z_{2ref} \tag{20}$$

We define a stable linear dynamics for this error:

$$\dot{\overline{z}}_2 = \dot{z}_2 - \dot{z}_{2ref} = v - \dot{z}_{2ref} \doteq -K_1(z_2 - z_{2ref}) \tag{21}$$

$$v = \dot{z}_{2ref} - K_1(z_2 - z_{2ref}) \tag{22}$$

$$\xi_{ref} = \alpha(z) + \beta(z)(\dot{z}_{2ref} - K_1(z_2 - z_{2ref}) \tag{23}$$

The error of ξ has the following form:

$$\overline{\xi} = \xi - \xi_{ref} \tag{24}$$

And so we define:

$$\dot{\overline{\xi}} = \dot{\xi} - \dot{\xi}_{ref} = -K_2\overline{\xi} \tag{25}$$

$$= a(\xi) + b(\xi)u - \frac{d}{dt}\left[\alpha(z) + \beta(z)(\dot{z}_{2ref} - K_1(z_2 - z_{2ref}))\right] = a(\xi) + b(\xi)u -$$

$$\frac{\delta\alpha(z)}{\delta z}\dot{z} + \frac{\delta\beta(z)}{\delta z}\dot{z}(z_{2ref} - K_1(z_2 - z_{2ref})) + \beta(z)(\ddot{z}_{2ref} - K_1(\dot{z}_2 - \dot{z}_{2ref})) = -K_2(\xi - \xi_{ref}) \tag{26}$$

An so we get the following expression for u:

$$u = \frac{1}{b(\xi)}\left[\frac{\delta\alpha(z)}{\delta z}\dot{z} + \frac{\delta\beta(z)}{\delta z}\dot{z}(z_{2ref} - K_1(z_2 - z_{2ref})) + \beta(z)(\ddot{z}_{2ref} - K_1(\dot{z}_2 - \dot{z}_{2ref})\right]$$

$$+\frac{1}{b(\xi)}\left[\beta(z)(\ddot{z}_{2ref} + K_1(\dot{z}_2 - \dot{z}_{2ref})) - a(\xi) + K_2(\xi - \xi_{ref})\right] \tag{27}$$

If we want to track a reference signal with the joint angle (α), and not with the joint angle velocity (ω), the following method for example can be a solution:

4.1 Trajectory design in ω

If we have a reference signal $\alpha_{ref}(t)$ and $\alpha(0)$ starting condition, we can define the trajectory:

$$\alpha_1(t) = \alpha_{ref}(t) + e^{-Ct}(\alpha_{ref}(0) - \alpha(0))$$

where C is a constant, which can be tuned.

We can use the time-derivative of this expression for reference to ω, and the higher order derivatives for further derivatives of the reference signal.

4.2 Simulation results

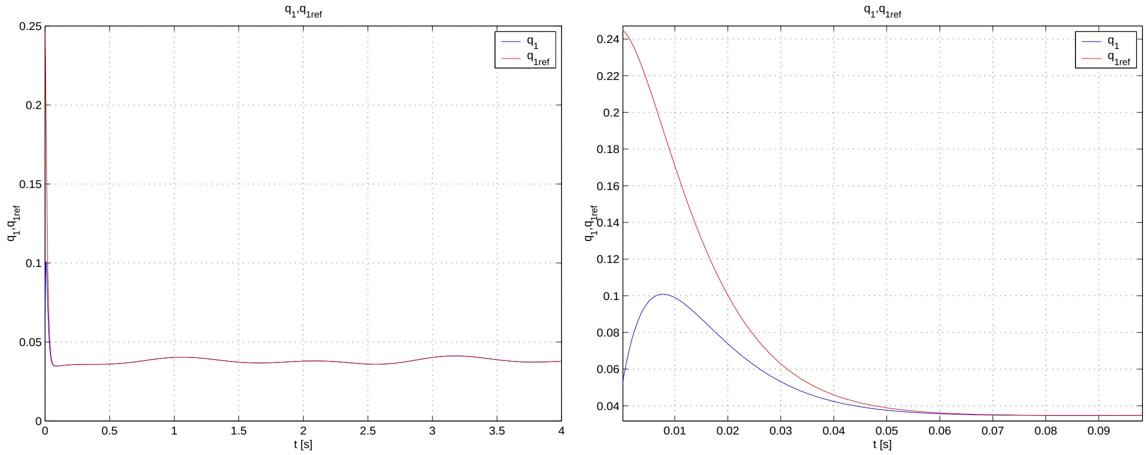

Figure 11: Muscle activation state, the reference signal for muscle activation during the whole movement and at the beginning

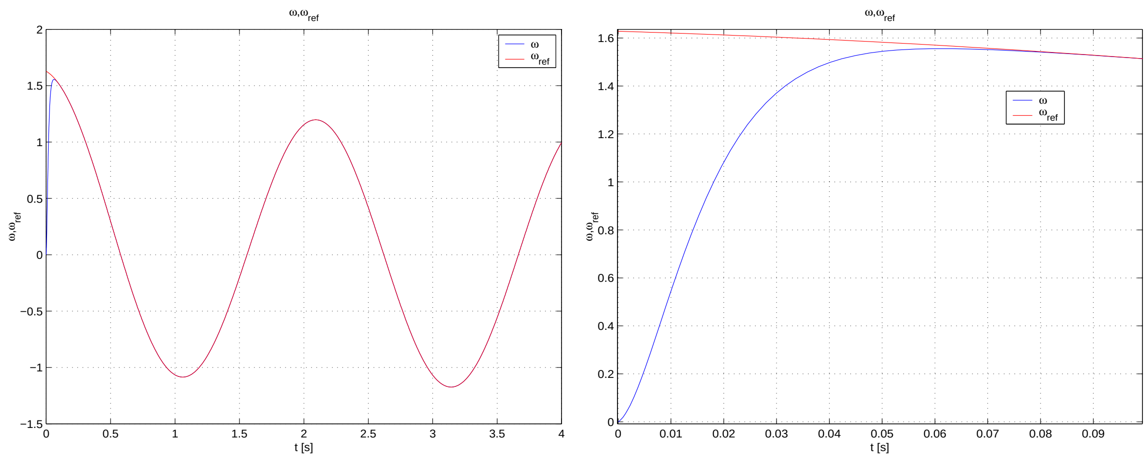

Figure 12: Joint angle velocity during the whole movement and at the beginning

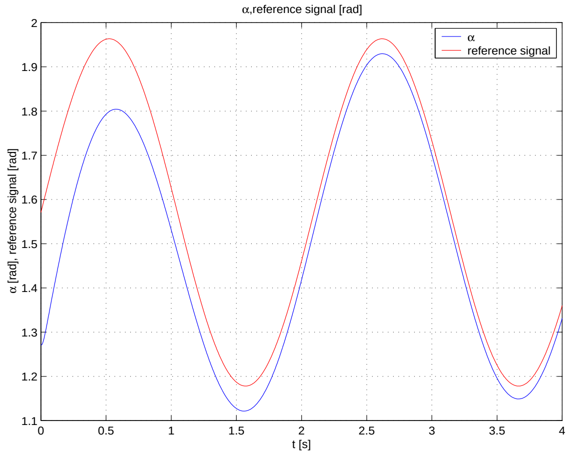

Figure 13: Output: Joint angle and the reference signal

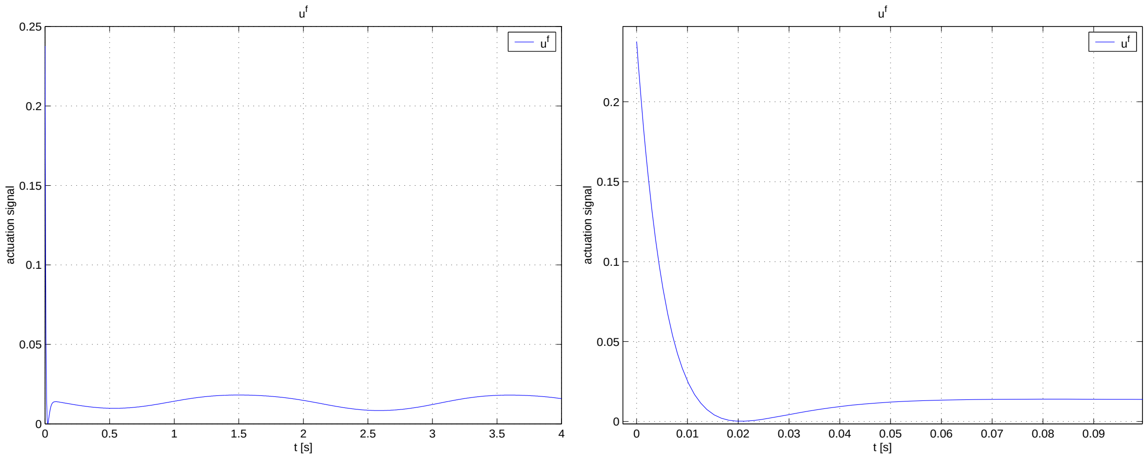

Figure 14: Input: Muscle activation signal during the whole movement and at the beginning

Because the joint angle velocity (ω) needs some time to reach the reference trajectory, a small remaining error can be seen in the joint angle reference tracking in figure 17. In fact the control performs even better if the starting conditions of ω and q_1 are closer to the reference trajectories:

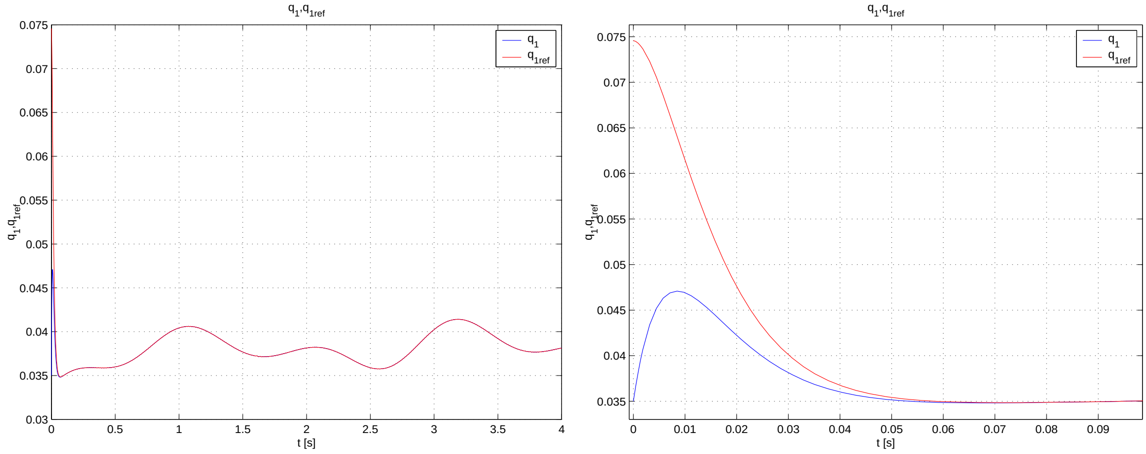

Figure 15: Muscle activation states, the reference signal for muscle activation during the whole movement and at the beginning

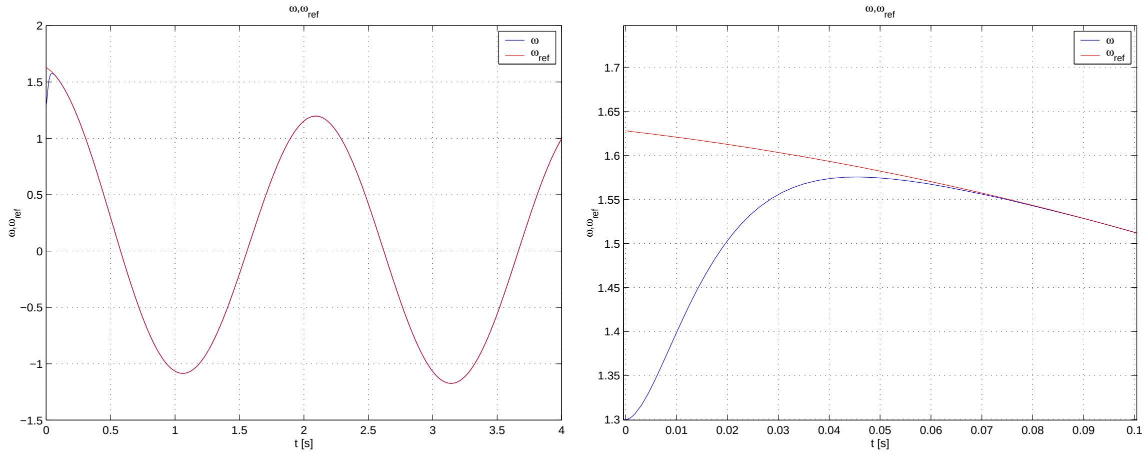

Figure 16: Joint angle velocity during the whole movement and at the beginning

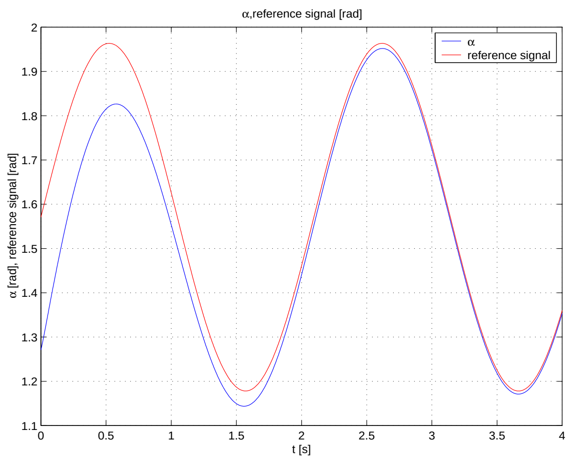

Figure 17: Output: Joint angle and the reference signal

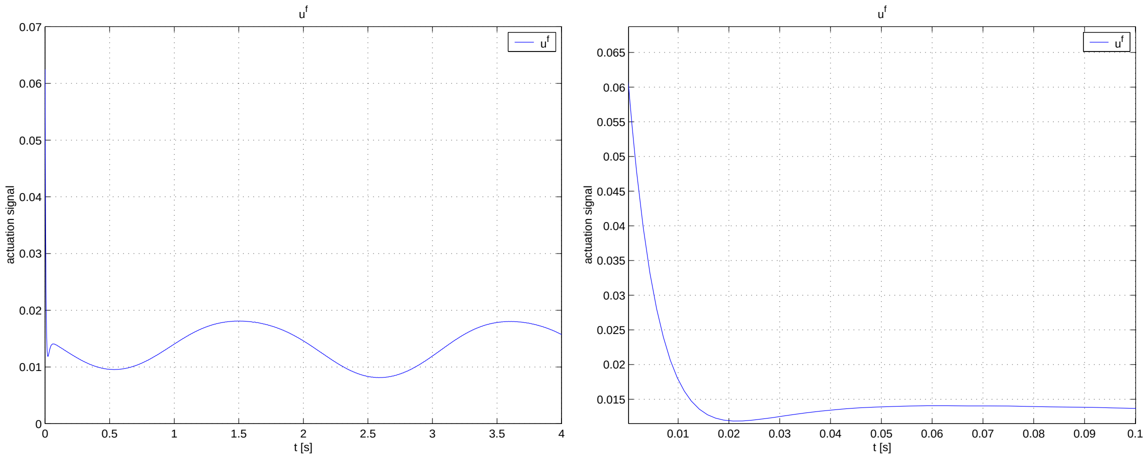

Figure 18: Input: Muscle activation signal during the whole movement and at the beginning

5 Conclusions and future work

We have shown that in the case of a simple nonlinear limb model utilizing of the cascade structure for control design can result in benefits of control performance, and for the closed-loop structure stability can be shown. The controllers have several constants which can be further tuned for better performance in the cases of various tasks.

The possible tasks for the future could be:

- Involve the input constraints to controller design.
- Involve some gamma-loop model in the mechanism of control as a load-estimator and corrector structure.
- Compare the state trajectories of the closed-loop system with trajectories recorded in the case of real movement patterns.
- Utilize both of the muscles for a MIMO control.

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