Nonlinear Zero Dynamics Analysis and Control of a Low Power Gas Turbine

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Abstract
Background. Gas turbines are highly nonlinear systems, where the widely applied linear controllers based on locally linearized models are of limited use.
Method of approach. A simple nonlinear dynamic model is developed that is in a quasi-polynomial form. Nonlinear stability and zero dynamics analysis is applied to find a controller structure for input-output linearization.
Results. The estimated stability neighborhood of the system with turbine inlet pressure held constant is 30-35% of the operating domain of the turbine. The servo controller for the rotational speed is asyptotically stable and robust against disturbances in the operating domain.
Conclusions. The proposed servo controller has better qualitative and quantitative properties than a servo controller based on a locally linearized model.
Keywords: gas turbine, quasipolynomial, Lotka-Volterra, zero dynamics, stability, Lyapunov, LMI, input-output linearization, nonlinear control, servo control

1. Introduction

Gas turbines are important and widely used prime movers in transportation systems. Besides this main application area, gas turbines are found in power systems where they are the main power generators [1]. Therefore the modelling and control of gas turbines is of great practical importance.

Control techniques applied for gas turbines are most often based on linear controllers. These controllers are mainly variants of linear quadratic (LQ) controllers, e.g. in [2,3]. An LQ servo controller is applied to track a reference signal in [4]. LQG/LTR technique [5] and robust control system design has also been performed [6] for gas turbines.

In nonlinear control, however, the streamline is the application of adaptive (e.g. in [7]) and adaptive predictive (e.g. in [8]) control approaches, but there is a big lack of "classical" state-space nonlinear control. As a rare exception, in [9] the equation of mass flowrate of fuel (as the control input of a simplified single input-single output model) is determined by a nonlinear method.

The basic approaches for stability analysis of general nonlinear systems are the stability analysis of the locally linearized model (by eigenvalue checking), or finding an appropriate Lyapunov function for the nonlinear model [10]. However, for autonomous system models in special forms as quasipolynomial (QP) or Lotka-Volterra (LV) form, there are additional stability results. There are available Lyapunov-function candidates for these system classes for investigating global stability [11].
In addition, local stability analysis can easily be performed. However, there is little known about estimating the stability region around a locally stable equilibrium point [12] even for QP models.

In order to apply nonlinear state-space model based control, one has to develop a relative simple yet powerful dynamic model that is able to describe the nonlinear dynamic behavior of the gas turbine. A strongly nonlinear state space description of a low power gas turbine has been developed based on first engineering principles in an earlier paper [13]. Because of nonlinearities, however, the nonlinear dynamic analysis (controllability, observability and stability analysis) of the developed model can only be performed with difficulty, or in some cases it can not be computed symbolically at all [14].

An advanced and nonlinear control Lyapunov-function based block-structured controller [15] has been proposed for the same gas pilot-plant turbine that is used as a case study in this paper. In a PhD thesis [16], this nonlinear controller is compared with an LQ servo controller, as a reference case known from the literature. As a result of the comparison it is pointed out that the system controlled by the nonlinear control Lyapunov-function based controller exhibits similar or better qualitative and quantitative behavior, than the system controlled by the LQ-servo controller. However, the design of the nonlinear control Lyapunov-function based controller included some key heuristically performed steps that were strongly specific to the pilot-plant gas turbine model. Therefore, the need to apply an alternative technique a nonlinear controller based on input-output linearization has also been identified [17].

The outline of this paper is as follows. First a simplified version of the nonlinear gas turbine model in [13] is developed that is in a QP-form. Then the local stability of the QP model of the low power gas turbine is investigated in the case when the turbine inlet total pressure is held constant with an appropriate input function. For stability analysis we use quadratic Lyapunov functions that enable us to estimate stability regions, as well. Moreafter the local stability of the turbine is analyzed when the rotational speed is held constant with an appropriate input function. At the end of the paper, we build a controller which forces the turbine to track a reference signal for the rotational speed.

2. Basic notions and tools

The general state-space representation of lumped process systems consists of a nonlinear input-affine state equation and an output equation [18]:

$$\frac{dx}{dt} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in R^n$, $u \in R^p$, $x \in R^r$ are the vectors of state, input and output variables, respectively, while $f \in R^n \rightarrow R^n$, $g \in R^n \rightarrow R^p$ and $h \in R^n \rightarrow R^r$ are smooth functions of x.

2.1. Process systems in QP and LV forms

The state equation of a wide class of input-affine process system models with $u=0$ or with $u=\phi(x)$ can be written in the so-called quasipolynomial (QP) form [19]:

$$\frac{dx_i}{dt} = x_i(\lambda_i + \sum_{j=1}^{m} A_{ij} z_j), \quad i = 1, \dots, n, \quad m \geq n$$

where the terms

$$z_j = \prod_{k=1}^{n} x_k^{B_{jk}}, \quad j = 1, \dots, m \tag{3}$$

are the so-called *quasimonomials (QMs)* of the system, A and B are real matrices of size *nxm* and *mxn* respectively, while λ is an *n*-dimensional real vector. Note that the domain of a QP-model is restricted to the positive orthant, i.e. $x_i > 0$, $i=1,...,n$.

It is a well-known fact that by differentiating the QMs by time we get a classical Lotka-Volterra (LV) representation, where the state variables are exactly the QMs [20]:

$$\frac{dz_\ell}{dt} = z_\ell(\lambda_{LV_\ell} + \sum_{j=1}^{m} A_{LV_{\ell,j}} z_j), \quad \ell = 1, \dots, m \tag{4}$$

where $\lambda_{LV} = B\lambda \in R^m$, $A_{LV} = BA \in R^{m \times m}$. Although LV models are one of the simplest nonlinear autonomous state space representations, LV models computed from QP systems are generally non-minimal with rank-deficient coefficient matrix A_{LV} (when $m>n$) which causes difficulties in stability analysis.

We also have to note that since all QMs depend on n variables, but the number of QMs are $m \geq n$, we can construct a basis of n QMs from which all other m-n QMs can be obtained. It means that we have additional m-n algebraic constraints between the state variables of the LV model, and therefore the system trajectories evolve on an n-dimensional manifold (surface) M of R^m.

2.2. Linear matrix inequalities

A (non-strict) linear matrix inequality (LMI) is an inequality of the form [21]:

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i \geq 0 \tag{5}$$

where $x \in R^m$ is the variable and $F_i \in R^{n \times n}$, $i=0,...m$ are given symmetric matrices. The inequality symbol in Eq. (5) stands for the positive semi-definiteness of $F(x)$. LMIs form a ***convex constraint*** on the variables i.e. the set $\{x | F(x) \geq 0\}$ is convex. Linear matrix inequalities have been playing an increasingly important role in the field of optimization and control theory since a wide variety of different problems (linear and convex quadratic inequalities, matrix norm inequalities, convex constraints etc.) can be written as LMIs and there are computationally stable and effective (polynomial time) algorithms for their solution [21,22].

2.3. Local quadratic stability of LV systems

Local stability of LV systems is investigated in this section with quadratic Lyapunov functions. At the end of this section, a method for investigating the stability region of non-minimal LV systems will be established.

Let us perform a coordinates shift on the LV-equations, i.e. $\underline{z} = z - z^*$, where z^* is a vector containing the operating point values of the LV variables, and therefore $\underline{z}$ is the centered LV variable vector. Then the LV-equations in Eq. (4) in the transformed coordinates have the form

$$\frac{d\overline{z}}{dt} = (\overline{Z} + Z^*) \cdot A_{LV} \cdot \overline{z}$$

where $\underline{Z}=diag(\underline{z}_1,...,\underline{z}_m)$, $Z^*=diag(z_1^*,...,z_m^*)$. Then, the quadratic Lyapunov function and its time derivative is in the following form:

$$V(\overline{z}) = \overline{z}^T P\overline{z}$$

$$\frac{dV}{dt} = \overline{z}^T P \frac{d\overline{z}}{dt} + \left(\frac{d\overline{z}}{dt}\right)^T P\overline{z} = \overline{z}^T \left(P\overline{Z}A_{LV} + PZ^* A_{LV} + A_{LV}^T \overline{Z}P + A_{LV}^T Z^* P \right) \overline{z}$$

where P is a positive definite symmetric matrix of size $m \times m$,
The non-increasing nature of the quadratic Lyapunov function in a neighborhood N of the origin is equivalent with the validity of the following matrix inequalitiy with variables $P=P^T>0$ *and* $\underline{Z}$::

$$P\overline{Z}A_{LV} + PZ^* A_{LV} + A_{LV}^T \overline{Z}P + A_{LV}^T Z^* P \le 0 \tag{6}$$

where $\underline{Z}=diag(\underline{z}_1,...,\underline{z}_m)$ and $(\underline{z}_1,...,\underline{z}_m)^T \in N$.
Therefore the quadratic stability region can be estimated by

- solving Eq. (6) for P with $\underline{Z}=0$,
- with this P finding all solutions of the LMI in Eq. (6) for $\underline{Z}$.

The solution set $\underline{Z}$ defines a a *convex neighbourhood of the origin* which is the estimated quadratic stability region.

2.3.1. Extension to non-minimal LV systems

The transformation of a QP model into LV form leads to a considerable increase of the state space dimension, since the number of the QMs is much higher than that of the QP variables in most of the cases. As a consequence, the LV system is non-minimal. Since the QMs depend on $n=dim(x)$ variables as defined in Eq. (3), the set of independent QMs z_{indep} contains n elements, and the remaining m-n QMs can be determined from the elements of z_{indep}. It means that we only have to determine n values in $\underline{Z}$ and then compute the other m-n values therefrom.

For investigating the local quadratic stability of non-minimal LV systems, a Theorem in [23] can be applied:

Theorem 1. *Let z^* be an equilibrium point of $dz/dt=f(z)$, and let $V:\chi \rightarrow R$ a C^1 function which is positive semidefinite at z^*, that is* $\boldsymbol{V(z^*)=0}$, $\boldsymbol{V(z)\geq 0}$. *Furthermore, suppose that $dV/dt \leq 0$ for all $z \in \chi$. Let K be the largest positively invariant set contained in $\{z|V(z)=0\}$. If z^* is asymptotically stable conditionally to K, then z^* is a stable equilibrium of $dz/dt=f(z)$.*

Let us consider the case when the eigenvalues of the matrix $J_{LV}=Z^*A_{LV}$ correspond to Jordan blocks of size 1. Then we can find a similarity transformation T such that

$$\tilde{J}_{LV} = T^{-1} J_{LV} T = diag(J_{S_{n\times n}}, 0_{(m-n)\times(m-n)})$$

where $J_{S,n\times n}$ and $0_{(m-n)\times(m-n)}$ are quadratic matrices, $n=rank(J_{LV})$ and 0 contains only zero entries. If the eigenvalues of J_S have strictly negative real parts, then it fulfills a Lyapunov iinequality from which the solution in the original coordinates can be determined [22]:

$$\left\{ P \middle| J_{LV}^T P + PJ_{LV} \le 0,\ P > 0 \right\} = \left\{ T^{-T} \widehat{P} T^{-1} \middle| \begin{array}{c} \widehat{P} = diag(P_{stab}, P_r) \\ P_{stab} > 0,\ P_r = 0 \\ J_S^T P_{stab} + P_{stab} J_S \le 0 \end{array} \right\} \tag{7}$$

By that, we obtain a positive *semi*definite quadratic Lyapunov function candidate. According to *Theorem 1.* the asymptotic stability of z^* on the largest invariant set of the manifold M contained in $ker(P)$ (i.e. where $\underline{z}^T P \underline{z}=0$) indicates (local) stability of z^*.

2.4. Zero dynamics of QP systems with LTI input and linear output terms

Let us consider now a special class of Eqs. (1-2) with scalar input and output variables, where the state function $f(x)$ is in QP form, the input function $g(x)=b=[b_1...b_n]^T$ is a constant vector, and the output function $h(x)$ is a linear function of the state variables:

$$\frac{dx}{dt} = x_i(\lambda_i + \sum_{j=1}^{m} A_{ij} \prod_{k=1}^{n} x_k^{B_{jk}}) + b_i u, \quad i = 1,\dots,n, \quad m \ge n \tag{8}$$

$$y = C(x - x^*) = \sum_{i=1}^{n} c_i (x_i - x_i^*) \tag{9}$$

where $A \in R^{n \times m}$ and $B \in R^{m \times n}$ are constant matrices, $\lambda \in R^n$ is a constant vector, and the vector x^* contains the steady state values of x.

The zero-output constrained dynamics (zero dynamics) of a system is the original system with the constraint $y \equiv 0$ [10]. Zero dynamics can be used to choose control structure of single input-single output systems.

In order to set the output to be identically zero we have to choose an appropriate input function u, which can be determined from the time-derivative of y (i.e. from the equation $dy/dt=0$), if the constant condition

$$\sum_{\ell=1}^{n} c_\ell x_\ell \neq 0 \tag{10}$$

is fulfilled. This is the case when the relative degree (r) of the system (the number of times the output has to be differentiated at $t=t_0$ in order to have the input value $u=u(t_0)$ explicitly appearing) equals to one ($r=1$).

Note that the relative degree is a local property in general, but because of the LTI input and linear output terms, the $r=1$ case is a global property of this model class (if the condition in Eq. (10) is fulfilled).

Since (if Eq. (10) holds) the expressed zeroing input is a polynomial function of x, the zero dynamics of Eqs. (8-9) is an *autonomous QP system*.

3. Modelling and model simplification of the low power gas turbine

In this section, we develop a dynamic model of the gas turbine, based on first engineering principles. The developed model is in input-affine form, but because of its strong nonlinearities it is further simplified in two steps.

3.1. Dynamic model of the gas turbine

The main parts of a gas turbine include the inlet duct, the compressor, the combustion chamber, the turbine and the nozzle or the gas-deflector (Fig. 1.). The interactions between these components are fixed by the physical structure of the engine. The operation of all types of gas turbines is basically the same. The air is drawn into the engine through the inlet duct by the compressor, which compresses it and then delivers it to the combustion chamber. Within the combustion chamber the air is mixed with fuel and the mixture is ignited, producing a rise in temperature and hence an expansion of the gases. These gases are exhausted through the engine nozzle or the engine gas-deflector, but first pass through the turbine, which is designed to extract sufficient energy from them to keep the compressor rotating, so that the engine is self sustaining.

3.1.1. Modelling assumptions

In order to get a low order dynamic model suitable for control purposes the following modelling assumptions should be made.

General assumptions

- Constant physico-chemical properties are assumed in each main part of the gas turbine, such as specific heat at constant pressure and at constant volume, specific gas constant and adiabatic exponent.
- Heat loss (heat transmission, heat conduction, heat radiation) is neglected.

Other assumptions

- In the inlet duct a constant pressure loss coefficient (σ_I) is assumed.
- In the inlet and in the outlet of the compressor the mass flow rates are the same: $\nu_{Cin}=\nu_{Cout}=\nu_C$, and there is no energy storage effect: $U_2=constant$.
- In the combustion chamber constant pressure loss coefficient (σ_{Comb}) and constant efficiency of combustion (η_{Comb}) are assumed; the enthalpy of fuel is neglected, and the combustion chamber is assumed to be a perfectly stirred region (balance volume). It means that a finite dimensional concentrated parameter model is developed and the value of the variables within this balance volume is equal to that at its outlet.
- In the inlet and in the outlet of the turbine the mass flow rates are the same: $\nu_{Tin}=\nu_{Tout}=\nu_T$, and there is no energy storage effect: $U_4=constant$.
- In the gas-deflector a constant pressure loss coefficient (σ_N) is assumed.

3.1.2. Conservation balances

The nonlinear state equations are derived from the laws of conservation principles [24]. Dynamic equations come from the conservation balances constructed for the overall mass m and internal energy U [18]. The development of the model equations is performed in the following steps.

Conservation balance of the total mass:

$$\frac{dm}{dt} = v_{in} - v_{out}$$

Conservation balance of the internal energy, where the heat energy flows and the power terms are also taken into account:

$$\frac{dU}{dt} = v_{in} i_{in} - v_{out} i_{out} + Q + P$$

We can transform the above energy conservation equation to its intensive variable form by considering the dependence of the internal energy on the measurable temperature:

$$\frac{dU}{dt} = c_v \frac{d}{dt}(Tm) = c_v T \frac{dm}{dt} + c_v m \frac{dT}{dt}$$

From the two equations above we get a state equation for the temperature as state variable:

$$\frac{dT}{dt} = \frac{v_{in} i_{in} - v_{out} i_{out} + Q + P - c_v T \ (v_{in} - v_{out})}{c_v m}$$

The ideal gas equation *(pV=mRT)* is used together with the two balance equations above to develop an alternative state equation for the pressure:

$$\frac{dp}{dt} = \frac{R}{c_v V}\left(v_{in} i_{in} - v_{out} i_{out} + Q + P\right)$$

Conservation balance of the mechanical energy of the compressor-turbine shaft:

$$\frac{dE_{shaft}}{dt} = v_T c_{pgas} \left(T_3 - T_4\right) \eta_{mech} - v_C c_{pair} \left(T_2 - T_1\right) - 2\pi \frac{3}{50} M_{load}$$

Note that the quantities *T, p* and *i* are ***total*** parameters of the stream.

3.1.3. Conservation balances in intensive form

These dynamic equations have to be transformed into their intensive variable form to contain measurable quantities. Therefore the set of transformed differential balances include the dynamic mass balance for the combustion chamber, the pressure form of the state equation derived from the internal energy balance for the combustion chamber and the intensive form of the overall mechanical energy balance expressed for the rotational speed *n* [25].

$$\frac{dm_{Comb}}{dt} = v_C + v_{fuel} - v_T \tag{11}$$

$$\frac{dp_3}{dt} = \frac{R_{med}}{c_{vmed} V_{Comb}} \left(v_C c_{pair} T_2 - v_T c_{pgas} T_3 + Q_f \eta_{Comb} v_{fuel} \right) \tag{12}$$

$$\frac{dn}{dt} = \frac{1}{4\pi^2 \Theta n} \left(v_T c_{pgas} \left(T_3 - T_4\right) \eta_{mech} - v_C c_{pair} \left(T_2 - T_1\right) - 2\pi \frac{3}{50} n M_{load} \right) \tag{13}$$

3.1.4. Constitutive (algebraic) equations

Some constitutive equations are also needed to complete the nonlinear gas turbine model [13].

1. Two equations come from the modelling assumptions for the total pressures after the compressor and the turbine:

$$p_2 = \frac{p_3}{\sigma_{Comb}} \qquad p_4 = \frac{p_1}{\sigma_I \sigma_N} \tag{14}$$

2. Ideal gas equation is used for the combustion chamber:

$$T_3 = \frac{p_3 V_{Comb}}{m_{Comb} R_{med}} \tag{15}$$

3. The third type of constitutive equations describes the total temperature after the compressor (T_2), and the total temperature after the turbine (T_4).
   - The total temperature after the compressor is found by using the isentropic efficiency of the compressor η_C in the following manner:

$$T_2 = T_1 \left(1 + \frac{1}{\eta_C}\left(\left(\frac{p_2}{p_1}\right)^{\frac{\kappa_{air}-1}{\kappa_{air}}} - 1\right)\right) \tag{16}$$

   - The total temperature after the turbine is found similarly by using the isentropic efficiency of the turbine η_T:

$$T_4 = T_3 \left(1 - \eta_T \left(1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{\kappa_{gas}-1}{\kappa_{gas}}}}\right)\right) \tag{17}$$

4. The fourth type of constitutive equations describes the mass flow rate and the isentropic efficiency of the compressor and the turbine.

$$v_C = const(1)q(\lambda_1)\frac{p_1}{\sqrt{T_1}} \quad , \quad v_T = const(2)q(\lambda_3)\frac{p_3}{\sqrt{T_3}} \tag{18}$$

   In the equations above $q(\lambda_1)$ and $q(\lambda_3)$ (the dimensionless mass flow rate of the compressor and the turbine) can be calculated as follows:

$$q(\lambda_1) = f_1\left(\frac{n}{\sqrt{\frac{T_1}{288.15K}}}, \frac{p_2}{p_1}\right) \tag{19}$$

$$q(\lambda_3) = f_2\left(const(3)\frac{n}{\sqrt{T_3}}, \frac{p_3}{p_4}\right) \tag{20}$$

   In the equations above $q(\lambda_1)$ is the function of the corrected rotational speed and the compressor pressure ratio, and $q(\lambda_3)$ is the function of the dimensionless velocity and the turbine pressure ratio.
   The equations of the isentropic efficiencies of the compressor and the turbine can be calculated as follows:

$$\eta_C = g_1\left(\frac{n}{\sqrt{\frac{T_1}{288.15K}}}, q(\lambda_1)\right) \tag{21}$$

$$\eta_T = g_2\left(const(3)\frac{n}{\sqrt{T_3}}, \frac{p_3}{p_4}\right) \tag{22}$$

The unknown static parameters of the dynamic model are the unknown coefficients of the polynomials approximating the characteristics of the compressor and the turbine, which can be identified by the least squares method using static measurements. The dynamic parameters of the model (V_{Comb}, Θ) can be identified by using measured step response functions and a simulation model containing the unknown parameters in a nonlinear form [14].

All constitutive equations are substitutable to the differential equations, so the final form of the nonlinear dynamic model of the investigated gas turbine is in the form of three ordinary differential equations obtained by substituting Eqs. (14-22) to Eqs. (11), (12) and (13). The verification of the dynamic model is performed by open-loop simulations.

3.1.5. The structure of the dynamic model in input affine form

The dynamic equations can be transformed into standard input-affine form of Eqs. (1-2) with the following *f*, *g* and *h* functions:

$$f(x) = \begin{pmatrix} f_1(x_1, x_2, x_3, d_1, d_2) \\ f_2(x_1, x_2, x_3, d_1, d_2) \\ f_3(x_1, x_2, x_3, d_1, d_2, d_3) \end{pmatrix}, \quad g(x) = \begin{pmatrix} b_1 \\ b_2 \\ 0 \end{pmatrix}, \quad h(x) = \begin{pmatrix} h_1(x_1, x_2, x_3, d_1) \\ x_2 \\ x_3 \end{pmatrix}$$

where the state (*x*), input (*u*), output (*y*) and disturbance (*d*) variables are:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} m_{Comb} \\ p_3 \\ n \end{pmatrix}, \quad u = v_{fuel}, \quad y = \begin{pmatrix} T_4 \\ p_3 \\ n \end{pmatrix}, \quad d = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} p_1 \\ T_1 \\ M_{load} \end{pmatrix}$$

It is important to note that these functions do not only depend on the state variables, but also on the disturbance vector. Further observe, that $g(x)$ does not depend on the state vector x (b_1 , b_2 are real constants). This means, that the effect of the input is linear to the time derivative of the state vector.

3.2. Model simplification

The coordinate functions of the state equation $f(x)$ have very complicated algebraic form, while the functions $g_i(x)$ are constants. Because of this fact, the nonlinear dynamic analysis of the model can only be performed with difficulty or in some cases it can not be done [14] analytically at all.

In order to be able to investigate the dynamic properties of the model and base controller design thereon, the nonlinear model has to be simplified.

3.2.1. Simplification assumptions

In order to obtain a feasible model form, additional model simplification assumptions are applied to the above model that are valid in the original operating region.

- In the first step, the isentropic efficiencies of the compressor and the turbine are assumed to be constants, because they slightly change their values within the operating region. Their values are

$$\eta_C = 0.67585 \quad , \quad \eta_T = 0.85677$$

- The second simplifying modelling assumption is that the physico-chemical properties (such as specific heats at constant pressure and at constant volume, specific gas constants and adiabatic exponents) of air - in the compressor - and hot gas - in the turbine - are assumed to have the same values.

$$c_p = c_{pair} = c_{pgas} = 1004.5\ J/kgK \quad \kappa = \kappa_{air} = \kappa_{gas} = 1.4$$
$$c_v = c_{vair} = c_{vgas} = 717.5\ J/kgK \quad R = R_{air} = R_{gas} = 287\ J/kgK$$

3.2.2. The simplified model

Consider the set of possible disturbances to be constants with their typical values:

$$d = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} p_1 \\ T_1 \\ M_{load} \end{pmatrix} = \begin{pmatrix} 101189.15\ Pa \\ 288.15\ K \\ 50\ Nm \end{pmatrix}$$

The above two simplification assumptions have been applied to the model equations in Eqs. (11-13) with all the algebraic equations (Eqs. (14-22)) substituted and the following simplified model equations have been obtained:

$$\frac{dx_1}{dt} = x_1(c_{1,1}x_1^{-1}x_2x_3 + c_{1,2}x_1^{-1}x_3 + c_{1,3}x_1^{-1}x_2 + c_{1,4}x_1^{-1} + c_{1,5}x_2x_3 + c_{1,6}x_3 + $$
$$+ c_{1,7}x_1^{-0.5}x_2^{1.5} + c_{1,8}x_1^{-0.5}x_2^{0.5}) + b_1u = f_1(x_1, x_2, x_3) + b_1u \tag{23}$$
$$\frac{dx_2}{dt} = x_2(c_2 + c_{2,1}x_3 + c_{2,2}x_2^{0.2857}x_3 + c_{2,3}x_2^{-1}x_3 + c_{2,4}x_2^{-0.7143}x_3 + c_{2,5}x_2^{0.2857} +$$

$$+ c_{2,6}x_2^{-1} + c_{2,7}x_2^{-0.7143} + c_{2,8}x_2x_3 + c_{2,9}x_1^{-0.5}x_2^{1.5} + c_{2,10}x_1^{-0.5}x_2^{0.5}) + b_2u =$$
$$= f_2(x_1, x_2, x_3) + b_2u \qquad (24)$$

$$\frac{dx_3}{dt} = x_3(c_{3,1}x_2^2x_3^{-1} + c_{3,2}x_2^{1.7143}x_3^{-1} + c_{3,3}x_2x_3^{-1} + c_{3,4}x_2^{0.7143}x_3^{-1} + c_{3,5}x_1^{-0.5}x_2^{2.5}x_3^{-2} +$$
$$+ c_{3,6}x_1^{-0.5}x_2^{2.2143}x_3^{-2} + c_{3,7}x_1^{-0.5}x_2^{1.5}x_3^{-2} + c_{3,8}x_1^{-0.5}x_2^{1.2143}x_3^{-2} + c_{3,9}x_3^{-2} +$$
$$+ c_{3,10}x_2^{1.2857}x_3^{-1} + c_{3,11}x_3^{-1} + c_{3,12}x_2^{0.2857}x_3^{-1} + c_{3,13}x_2x_3^{-2} + c_{3,14}x_2^{1.2857}x_3^{-2} +$$
$$+ c_{3,15}x_2^{0.2857}x_3^{-2}) = f_3(x_1, x_2, x_3) \qquad (25)$$

where $u=v_{fuel}$ is the scalar valued control input, $x=[x_1\ x_2\ x_3]^T=[m_{Comb}\ \ p_3\ \ n]^T$ is the vector of state variables, and $c_{i,j}$'s and b_i 's are the parameters of the model. The relationship between the parameters of the original engineering model and that of the above simplified model in QP-form is given in Table 2. in the Appendix.

The parameters of the simplified model have been estimated using measured data following the method described in [14], their estimated values together with their units can be found in Table 3. in the Appendix.

The simplified dynamical model of the gas turbine is valid within the following operating domain:

$$0.00305\, kg = x_{1\min} \le x_1 \le x_{1\max} = 0.00835\, kg$$
$$154837\ Pa = x_{2\min} \le x_2 \le x_{2\max} = 325637\ Pa$$
$$650\ \ 1/s = x_{3\min} \le x_3 \le x_{3\max} = 833.33\ \ 1/s$$

The most important notations are explained in Table 1., while a comprehensive list is given in the Nomenclature.

Table 1: Notations

Not.	Variable name / Units
m_{Comb}	mass in combustion chamber *[kg]*
P_3	turbine total inlet pressure *[Pa]*
N	rotational speed *[1/s]*
v_{fuel}	mass flowrate of fuel *[kg/s]*
P_1	compressor inlet total pressure *[Pa]*
T_1	compressor inlet total temperature *[K]*
M_{load}	loading moment *[Nm]*
T_4	turbine outlet total temperature *[K]*

With these simplifications *the resulted dynamic model in Eqs. (23-25) has QP state and LTI input terms*, i.e. it is in the form of Eq. (8).

In the operating region, for the constant reference control input value $u^*=0.009913\ kg/s$ there is a unique steady state point: $x^*=[0.00528\ kg\ \ \ 223587\ Pa\ \ \ 750\ 1/s]^T$ that has been chosen for our stability investigations.

4. Local stability of the turbine model with inlet total pressure held constant

In this section the zero dynamics of the system for turbine inlet total pressure held constant will be computed first, and local stability analysis is then applied by computing the quadratic stability region, as well. At the end of the section, simulation results are presented.

4.1. Zero dynamics for turbine inlet total pressure

Let us consider the case when the pressure p_3 is held constant $(x_2=x_2^*=p_3^*)$ with an appropriate control input. This control goal can be re-phrased as finding a "zeroing input" for the artificial output $y_{ZD}=x_2-x_2^*$, e.g. the zero dynamics of the model for the output $y_{ZD}\equiv 0$. Since the relative degree equals to one in this case with the LTI input, the zeroing input can be computed easily.

The zero dynamics of the QP gas turbine model is then an autonomous QP system with two state variables and eight QMs. The parameters of the LV model are $A_{LV}=BA\in R^{8\times 8}$ and $\lambda_{LV}=B\lambda\in R^8$. Choosing $\{z_1=x_{1d}^{-1}, z_2=x_{3d}\}$ as a base, where the subscript denotes *dimensionless* variables, all the LV-variables can be computed, defining the manifold *M* where the state trajectories can evolve.

4.2. Local quadratic stability

Since A_{LV} is rank-deficient, the method in Section 2.3.1. is applied for quadratic stability analysis. The quadratic Lyapunov function computed by Eq. (7) is $V(\underline{z})=\underline{z}^TP\underline{z}$, where *P* is positive *semi*definite and fulfills the LMI in Eq. (6) at the origin $\underline{z}=0$ (i.e. where $z=z^*$).

Numerical computations showed that $V=0$ is fulfilled in two isolated points on the state manifold M: one of them is the operating point, the other is at ($m_{Comb}=0.3577$ kg, $n=452$ 1/s), which is outside of the valid operating domain of the turbine model. It means that in the valid operating domain of the turbine, the largest positively invariant set in *Theorem 1.* is $K=\{z^*\}$. Thus, the operating point z^* is locally asymptotically stable with respect to K showing that z^* *is a (locally) stable equilibrium of* R^8.

4.2.1. Estimation of quadratic stability region

Since LMIs form convex constraints on their variables, we only have to solve Eq. (6) for $\underline{Z}$ to find a convex stability region of the origin. Note that $z_3,..., z_8$ are determined by z_1 and z_2, moreover the (nonstrict) feasibility of this LMI depends on these two variables only (because of the rank deficit in the LMI causing six zero eigenvalues). Therefore we only have to change z_1 and z_2 to find a convex region in R^2 where the LMI in Eq. (6) is valid.

In our case, this convex set is a quadrangle depicted in Fig. 2. (dashed line). The operating domain (i.e. where the model is valid) is a box (solid line) containing the operating point (denoted by a '+').

A quadratic stability neighborhood is an ellipsoid determined by the level-curves of $\underline{z}^T P \underline{z}=c$ where c is a real constant. This ellipsoid is eight-dimensional in our case. Fortunately, we only have to consider it in the two-dimensional state manifold M. Since $z_3,..., z_8$ are determined by z_1 and z_2 we substitute them into $\underline{z}^T P \underline{z}=c$ and get a nonlinear inplicit relationship between z_1 and z_2. This equation gives closed Lyapunov level-curves. The biggest level curve fitting in the convex region surrounds the quadratic stability neighborhood as it is depicted in Fig. 2. The area of the stability neighborhood is approximately 30-35 percents of the operating domain.

5. Zero dynamics, input-output linearization and servo control of the turbine with the rotational speed held constant

In this section we consider the case when the rotational speed is held constant with an appropriate control input function. Having investigated the stability of this zero dynamics, input-output linearization and LQ servo control based on the input-output linearized model is also described.

5.1. Zero dynamics of the turbine with the rotational speed held constant

The control input which holds the rotational speed on a constant reference value is exactly the zeroing input for the zero dynamics of the simplified system model in Section 3.2.2. with the output $y_{ZD}=x_3-x_3^*$. The input does not appear directly in the time-derivative of the output, because the third differential equation, Eq. (25) does not contain any input term. It means that the relative degree is greater than one in this case. Differentiating y_{ZD} twice by time we get

$$\frac{d^2 y_{ZD}}{dt^2} = \frac{\partial f_3}{\partial x}\frac{dx}{dt} = \frac{\partial f_3}{\partial x_1} f_1(x) + \frac{\partial f_3}{\partial x_2} f_2(x) + \frac{\partial f_3}{\partial x_3} f_3(x) + \left(b_1 \frac{\partial f_3}{\partial x_1} + b_2 \frac{\partial f_3}{\partial x_2} \right) u \tag{26}$$

It means that $r=2$ in a *neighborhood* of the operating point $N(x^*)$, where the condition

$$b_1 \frac{\partial f_3}{\partial x_1} + b_2 \frac{\partial f_3}{\partial x_2} \neq 0, \quad x \in N(x^*)$$

is fulfilled. It also means that the zero dynamics is one dimensional, since $y_{ZD} \equiv 0$ and $dy_{ZD}/dt \equiv 0$ defines two algebraic constraints on the state variables.

We use the traditional method for computations. First we express the zeroing input from Eq. (26). Substituting the zeroing input and the two algebraic constraints ($x_3=0$ and $x_1=\phi(x_2)$) to Eq. (24) we get the one-dimensional zero dynamics which is an autonomous differential equation:

$$\frac{dx_2}{dt} = F(x_2) \tag{27}$$

Note that now the zeroing input is *not in QP form*, because it is the ratio of two polynomials. From this fact it comes that the zero dynamics will not be in QP form.

Since $F(x_2)$ is *not in QP form*, the method that has been used formerly cannot be applied here. Due to the extreme complexity of $F(x_2)$, even nonlinear analysis methods are especially hard to be applied.

Thus, Eq. (27) is a differential equation of one variable, therefore the stability neighborhood of the equilibrium point x_2^* can be determined from the phase diagram (the graph that describes the dependence of $F(x_2)=dx_2/dt$ on x_2) depicted in Fig 4. This diagram shows that in the valid operating domain of the turbine, the only equilibrium point is x_2^* which is stable in this domain (the phase diagram has a negative slope).

The local stability of this one dimensional zero dynamics is a key fact in controller design, because its stability is compulsory for the stability of the controlled plant. Note that the stability of this zero dynamics has been checked at several different steady state operating points which belong to different steady state reference control input values, and it is found to be stable in all cases. This indicates our hypothesis that this dynamics is locally asymptotically stable independently of steady state values.

5.2. Input-output linearization

In this section, input-output linearization is performed for the rotational speed of the turbine in order to add a linear servo controller. The (input-output) linearized model enables that the turbine is asymptotically stabilized with a simple linear quadratic (LQ) controller.

The flow diagram of an input-output linearized plant is depicted in Fig. 5. The plant S is nonlinear in itself, with control input u and output y. Then, with an appropriately chosen controller (denoted by C) the states x of the nonlinear plant are fed back in such a way, that the plant S^* with external input v and output y is *linear time invariant in input-output sense*.

In the previous section it was shown that the relative degree of the turbine model with output being the (centered) rotational speed is $r=2$ in a neighborhood of the operating point, therefore the (nonlinear) zero dynamics is one dimensional. Since the zero dynamics does not depend on the control input variable u, the remainder two-dimensional part of the model can be linearized - in input-output sense - by an appropriate control input function [10]. For this purpose, we turn back to the observation that the input appears in the second time-derivative of the rotational speed (x_3) first. Consider the following nonlinear feedback which contains a zeroing input with an additional external input function v (see Fig. 5.):

$$u = -\frac{\frac{\partial f_3}{\partial x_1} f_1(x) + \frac{\partial f_3}{\partial x_2} f_2(x) + \frac{\partial f_3}{\partial x_3} f_3(x)}{b_1 \frac{\partial f_3}{\partial x_1} + b_2 \frac{\partial f_3}{\partial x_2}} + \frac{1}{b_1 \frac{\partial f_3}{\partial x_1} + b_2 \frac{\partial f_3}{\partial x_2}} \mathrm{v}$$

Applying this input function to the equations of the simplified model (Eqs. (23-25)) we obtain that $d^2x_3/dt^2=\mathrm{v}$, therefore the input-output linearized system can be represented by the following state-space model:

$$\frac{d}{dt}\dot{\bar{x}}_3 = \mathrm{v} \tag{28}$$

$$\frac{d}{dt}\bar{x}_3 = \dot{\bar{x}}_3 \tag{29}$$

$$\frac{d}{dt}w = q(\bar{x}_3, \dot{\bar{x}}_3, w) \tag{30}$$

$$y = \bar{x}_3 \tag{31}$$

where q is the state function of the one dimensional zero dynamics. Since the output function y is not affected by the zero dynamics, moreover the external input function v does not influence it directly, the system is *linear in external (input-output) sense*. It is important to note that the state function q with complex algebraic form is *not needed* when computing the control feedback but only for analyzing the stability of the closed-loop system.

The transfer function of the input-output linearized model equals to $H(s)=1/s^2$ since the *linearized* subsystem in Eqs. (28,29,31) is a cascade of integrators.

It is known from [10], that if the zero dynamics in Eq. (30) is locally asymptotically stable around its equilibrium w^*, and the external input is chosen in such a way that the linear subsystem in Eqs. (28-29) is locally asymptotically stable around its equilibrium point $[0\ \ 0]^T$, then the whole system in Eqs. (28-30) will be locally asymptotically stable around the equilibrium $[0\ \ 0\ \ w^*]^T$. It means that the plant can be asymptotically stabilized by an appropriate linear feedback.

5.3. Servo controller with stabilizing feedback

Our aim is to build a controller which tracks the reference signal v_{ref} which is our prescribed value for the rotational speed. For this purpose, we extend our model in Eqs. (38-41) with the following differential equation:

$$\frac{d}{dt}e = \mathrm{v}_{ref} - y = \mathrm{v}_{ref} - \bar{x}_3 \tag{32}$$

Where e is the tracking error, which is zero at steady state, meaning that $y=\mathrm{v}_{ref}$ (at steady state). Since Eq. (32) is a linear differential equation with constant coefficients, the linear subsystem in Eqs. (28-29) with this adjoint

makes an LTI system with state equations in Eq. (28,29,32) and output equation in Eq. (31). This system has two input variables: the external reference signal (v_{ref}) and v:=v_{stab} which will be the stabilizing feedback of the system as determined in the following.
where the external input contains the reference signal (v_{ref}) and a stabilizing feedback (v_{stab}) which will be determined in the following.

Since all the eigenvalues of the state matrix of the servo controlled plant are at the origin, we need an additional linear state feedback to asymptotically stabilize the system. We apply an LQ controller with state (Q) and input (R) weighting matrices in the following way: We fix R to the unit value, since only the relationship between the magnitude of elements of Q and R that matters. In the weighting matrix Q, we neglect the cross-effects between the state variables. Thus, we have three parameters to tune the controller, with respect to two goals: Let the settling time of the plant between *1.5 s* and *2 s*, and let the plant be robust against disturbances. The tuning of parameters is performed via simulations, and gives the weighting matrices $Q=diag(1000, 1500000, 5000000)$, $R=[1]$.
The stabilizing feedback computed in MATLAB is in the form $v_{stab}=[k_1\ k_2\ k_3][dx_3/dt\ x_3\ e]^T$.
With this stabilizing feedback, the state space model of the input-output linearized model with servo control input v_{ref} is the following:

$$\frac{d}{dt}\begin{bmatrix}\dot{\bar{x}}_3\\ \bar{x}_3\\ e\end{bmatrix}=\begin{bmatrix}-60.509 & -1330.6 & 2236.11\\ 1 & 0 & 0\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}_3\\ \bar{x}_3\\ e\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}v_{ref}$$

$$y=\begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}_3\\ \bar{x}_3\\ e\end{bmatrix}$$

It is easy to see that if $v_{ref}= x_3^*$ (i.e. when $de/dt=0$) then the zero dynamics of the I/O linearized model and the zero dynamics of the linearized model with servo controller is identical. But in our case, the rotational speed (x_3) tracks a reference signal v_{ref} therefore the local stability of zero dynamics in equilibrium points with arbitrary x_3^* is compulsory for the stability of the servo controlled plant. This property is not proved analytically because of the extreme complexity of the zero dynamics, but this hypothesis is checked experimentally by investigating the asymptotic behavior of several steady state points with different x_3^* values and it has been found that the zero dynamics is locally asymptotically stable for every possible reference input u^*.

5.4. Simulation results

The MATLAB/SIMULINK model of the gas turbine model with input-output linearizing, servo and stabilizing feedback has also been built in order to confirm theoretical results. In Fig. 6., the four subfigures show the time function of control input and measured output variables - v_{fuel} , T_4 , p_3 and n respectively -, near typical parameter values (i.e. with no disturbance). The rotational speed (solid line in the 4th subfigure) is started from $n=750$ *1/s* and tracks a piecewise linear reference signal (dashed line in the 4th subfigure). The settling time (the time after the rotational speed remains within ±5% of the prescribed constant value) is about $t_s=1.7$ *s*. Simulations show that the controlled plant is asymptotically stable and tracks the reference signal with linear transients.

Figure 7. shows the reference tracking transients of n of the system started from five different points in the state-space. Transients denoted by dashed lines are started from $(m^*_{Comb}, p_{3min}, n_{min})$ and $(m^*_{Comb} , p_{3min} , n_{min})$ $(m^*_{Comb} , p_{3max} , n_{max})$, while the ones denoted by dash-dot lines are started from $(m^*_{Comb} , p_{3min} , n_{max})$ and $(m^*_{Comb} , p_{3max} , n_{min})$. One transient is started from the steady state point (solid line). As this figure shows, all transients are stable and track the constant reference signal (dashed line). These simulations show the asymptotic stability of the controlled plant.

Simulations also showed that the plant is robust against the change of all disturbance variables. Figure 8. shows the change of the rotational speed (3rd subfigure) in case when the loading moment is changed to its maximal value in two steps (2nd subfigure). The initial value of the rotational speed is $n=750$ *1/s*. The reference value is $v_{ref}=800$ *1/s*, which is successfully tracked until $t=2$ *s*. In this period, the loading moment is set to its typical value $M_{load}=50$ *Nm*. At $t=2$ *s*, the load is set to *200 Nm*, at $t=4$ *s* it is set to *350 Nm* causing drops, but the controlled plant turns back to the prescribed reference value in a short time.
The necessary control input function is depicted in the 1st subfigure. Also note that the control input never reaches its saturation value ($u_{sat}= 0.03$ *kg/s*).

5.5. Discussion

In this section, a reference tracking controller has been described and analyzed for the gas turbine model. We have decided to control the rotational speed instead of the turbine inlet total pressure (that was investigated in

Section 4. of this paper) because the (servo-) control of the rotational speed is much more widespread in practice, moreover, this quantity is more suitable for measurements.

In the PhD thesis mentioned earlier [16], an LQ-servo controller has been built for the turbine model that also uses the rotational speed to control. However, this controller is designed by using a linearized state space model of the turbine at an operating point, therefore stability is only guaranteed in a neighborhood of this steady state operating point.

Since a linear controller applied to a nonlinear model can only guarantee local properties, it led us to apply input-output linearization first, and then feed back with a linear controller. The main drawback of the I/O linearization method is the extreme complexity of the linearizing input function in contrast with the simple linear feedback applied earlier.

On the other hand, the application of an LQ-servo controller for the input-output linearized model guarantees asymptotic stability on the whole operating domain of the plant, while with the method in [16] trajectories started far from the steady state cannot turn back to the operating point. Because of its local property, the controller based on a locally linearized model cannot defend the gas turbine against too high temperature and rotational speed.

It is important to note, however, that both of the two above mentioned LQ servo controllers showed robustness against disturbances in our simulation experiments.

6. Conclusions

In this paper, a nonlinear model of a low-power gas turbine has been built from first engineering principles and has been simplified using model simplification to a QP-form suitable for control purposes. The resulted model has QP state and LTI input terms.

With an appropriate quadratic Lyapunov-function candidate, the local stability of the turbine model with turbine total inlet pressure held constant is investigated after transforming it to LV form. A new method based on linear matrix inequalities is applied to estimate the quadratic stability region. The biggest level curve of the quadratic Lyapunov function inside the estimated convex region encircles its stability neighborhood, which covers about 30-35% of the operating domain of the turbine model. Simulations confirmed theoretical results, and trajectories started out of the estimated stability neighborhood showed that the range of stability is wider than the estimated one.

The zero dynamics model of the turbine with the rotational speed held constant is not in QP form, but it has a one-dimensional dynamics, therefore the stability neighborhood of the operating point could be estimated with a phase diagram and has been found to be asymptotically stable.

A controller based on input-output linearization has also been built to track a prescribed reference signal for the rotational speed. Simulations in MATLAB/SIMULINK showed that the controlled system has a linear and stable dynamics, moreover the controlled plant is robust against disturbances.

This controller has been compared with a previous controller which was designed using the linearized model of the turbine at an operating point, and found better in the sense that it guarantees asymptotic stability of the controlled plant in the whole operating domain.

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Nomenclature

Variables/Constants

A	area [m^2]
M	moment [*Nm*]
P	power [*W*]
Q	heat flow [*J/s*]=[*W*]
Q_f	lower thermal value of fuel [*J/kg*]
R	specific gas constant [*J/(kg K)*]
T	temperature [*K*]
U	internal energy [*J*]
V	volume [m^3]
C	specific heat [*J/(kg K)*]
I	specific enthalpy [*J/kg*]
M	mass [*kg*]
N	rotational speed [*1/s*]
P	pressure [*Pa*]
Q	dimensionless mass flow rate [-]

T	time [*s*]
tk_1, tk_2	coefficients of q_1 [*s*]
tk_3, tk_4	coefficients of q_1 [-]
tt_i, $i=1,..,4$	coefficients of q_3 [-]
β	specific parameter of air and gas [$K^{0.5}s/m$]
η	efficiency [-]
Θ	inertial moment [$kg\ m^2$]
κ	adiabatic exponent [-]
λ	dimensionless speed [-]
ν	mass flowrate [*kg/s*]
σ	pressure loss coefficient [-]
τ	turbine velocity coefficient [$K^{0.5}s$]
	Subscripts
0	inlet duct inlet
1	compressor inlet
2	compressor outlet
3	turbine inlet
4	turbine outlet
air	refers to air
C	refers to compressor
Comb	refers to combustion chamber
comb	refers to combustion
fuel	refers to fuel
gas	refers to gas
I	refers to inlet duct
in	Inlet
load	Loading
mech	Mechanical
med	refers to medium parameters
N	refers to gas deflector
out	Outlet
p	refers to constant pressure
schaft	refers to schaft
T	refers to turbine
v	refers to constant volume
	Superscripts
*	refers to steady state value

Appendix

Table 2: Coefficients of the simplified model

Table 3: Constants of the simplified model

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