Model predictive control for the hybrid primary circuit dynamics of a pressurized water nuclear power plant *

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Abstract

In this paper, a model predictive controller is developed for controlling the main primary circuit dynamics of pressurized water nuclear power plants during load-change transients. The hybrid model of the plant is successfully embedded into a non-hybrid discrete time LPV form. The designed controller is able to handle the hard constraints for the state and input variables while keeping the plant stable and producing satisfactory time-domain behavior.

1 Introduction

The paper describes a model predictive control scheme for the primary circuit system of the Paks Nuclear Power Plant (Paks NPP) located in Hungary. The Paks NPP was founded in 1976 and started its operation in 1981. The plant operates four VVER-440/213 type reactor units with a total nominal (electrical) power of 1860 MWs. About 40 percent of the electrical energy generated in Hungary is produced here. Considering the load factors, the Paks units belong to the leading ones in the world and have been among the top twenty-five units for years.

The main motivations behind the present work are the following. Firstly, due to the continuous reconstruction of the measurement equipment and the information infrastructure, more and more measurement data are available in good quality. This fact allowed us the control-oriented modeling and parameter identification of the primary circuit dynamics [11]. Secondly, the present control configuration of the plant is a distributed scheme, where the controllers are tuned individually. The current operation of the system in the neighborhood of the prescibed operating points is satisfactory, but studies and simulations show that the dynamic behavior during bigger transients mainly caused by load changes can be improved by applying a multivariable controller. Thirdly, another motivating fact is a previous work: the successful modeling, identification [11], controller design [10] and implementation of the pressure control loop in the primary circuits of units 1, 3 and 4 of the plant. Using this model-based design, the precise stabilization of the primary loop pressure was a key factor in the safe increase of the average thermal power of the units by approximately 1-2% in 2005.

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The main aim of this paper is to propose an integrated controller, which eliminates some imperfections of the present control architecture. This new controller belongs to the model predictive control (MPC) scheme. The MPC is an optimization based control method, where, assuming discrete time case, an open loop optimal control problem is solved in each sampling instant, using the actual state as initial state, and the first element of the obtained control sequence is applied to the plant. For the theoretical background of MPC see e.g. [7], [6]. The model predictive approach has several advantages: it is able to handle complex systems where off-line computation of the control law is difficult, and in contrast to other techniques it is able to handle hard constraints prescribed for the states and control inputs.

The paper is organized as follows. After the introduction in section 1, a nonlinear hybrid model of the plant is constructed. Section 2 contains the main control objectives. In section 2.A, the LMI-based MPC method proposed by Kothare et.al [5] is introduced, which is slightly modified in section 2.B. The original and the modified methods are applied and tested on the nonlinear system model by numerical simulations. The simulation results are presented and analyzed in section 4. The most important conclusions are summarized in section 5.

2 System Model

2.1 Overall system description

The liquid in the primary circuit is circulated at a high speed by powerful circulation pumps, and it is under high pressure in order to avoid boiling. The energy generated in the reactor is transferred by the primary circuit to the liquid in the steam generator making it boiling. The generated secondary circuit vapor is then transferred to the turbines.

Figure 1 shows the flowsheet of the primary circuit in Paks NPP, where the main equipments are the reactor, the steam generator(s), the main circulating pump(s), the pressurizer and their connections are depicted. The sensors that provide on-line measurements are also indicated in the figure by small full rectangles. The controllers are denoted by double rectangles, their input and output signals are shown by dashed lines.

2.2 Continuous time state-space model

The dynamic model of the process has been constructed using a systematic modeling approach proposed in [4]. The detailed modeling and model identification procedure has been described in [3]. The continous time state-space model of the system is the following

$$\frac{dN}{dt} = \frac{\beta}{\Lambda}\left(\rho_{max} - (p_1 v^2 + p_2 v + p_3)\right) N + S \quad (1)$$

$$\frac{dT_{PC}}{dt} = \frac{1}{c_{p,PC} M_{PC}}\Big[c_{p,PC} m_{in}\left(T_{PC,I} - T_{PC,CL}\right) + W_R - 6 \cdot K_{T,SG_1}(T_{PC} - T_{SG}) - W_{loss,PC}\Big] \quad (2)$$

$$\frac{dT_{SG}}{dt} = \frac{1}{c^L_{p,SG} M_{SG}}\Big[c^L_{p,SG} m_{SG} T_{SG,SW} - c^V_{p,SG} m_{SG} T_{SG} - m_{SG} E_{evap,SG} + K_{T,SG_2}(T_{PC} - T_{SG}) - W_{loss,SG}\Big] \quad (3)$$

Figure 1: The flowsheet of the primary circuit

$$\left.\frac{dT_{PR}}{dt}\right|_{m_{PR}>0} = \frac{1}{c_{p,PR}M_{PR}}\Big[c_{p,PC}m_{PR}T_{PC,HL} - c_{p,PR}m_{PR}T_{PR} - \\ -W_{loss,PR} + W_{heat,PR}\Big] \quad (4)$$

$$\left.\frac{dT_{PR}}{dt}\right|_{m_{PR}\leq 0} = \frac{1}{c_{p,PR}M_{PR}}\Big[-W_{loss,PR} + W_{heat,PR}\Big] \quad (5)$$

where $W_R = c_\psi N$. The measurable variables and constant parameters of the model are summarized in table 1. The abbreviations R, PC, SG, PR refer to the *reactor*, *primary circuit*, *steam generator*, and *pressurizer*, respectively.

The mass flow m_{PR} (which makes the system dynamics hybrid) from the primary circuit to the pressurizer (or backward) can be written as

$$m_{PR} = -V_{PC}^0 c_{\phi,1}\frac{dT_{PC}}{dt} \quad (6)$$

We assume that the variables m_{in}, $T_{PC,I}$, m_{SG} and $T_{SG,SW}$ are known and constant which is an acceptable approximation of reality from a control point of view. The control inputs are the rod position (v) and the heating power of the pressurizer ($W_{heat,PR}$) . Instead of v we introduce $\nu = (p_1v^2 + p_2v + p_3)N$ as a new control input, since eq. (1) depends linearly on ν. This can be done, since the polynomial $p(v) = p_1v^2+p_2v+p_3$ is monotonously increasing, thus invertible: $v = p^{-1}(\nu/N)$. The constraints prescribed for v can be transformed into equivalent

Identifier	Variable	Type	Identifier	Parameter	Unit
N	R neutron flux	s	(p_1, p_2, p_2)	control rod parameters	R
v	R control rod position	i	ρ_{max}	maximum reactivity	R
W_R	R reactor power	o	S	zero neutron flux	R
m_{in}	PC inlet mass flow rate	i	$c_{p,PC}$	specific heat	PC
$T_{PC,I}$	PC inlet temperature	d	M_{PC}	water mass	PC
$T_{PC,CL}$	PC cold leg temperature	(s)	$K_{T,SG_{1,2}}$	heat transfer coefficients	PC, SG
$T_{PC,HL}$	PC hot leg temperature	(s)	T_{out}	containment temperature	PC
p_{PR}	PR pressure	o,(s)	M_{SG}	water mass	SG
T_{PR}	PR temperature	s	$W_{loss,PC}$	heat loss	PC
ℓ_{PR}	PR water level	o,(s)	$W_{loss,SG}$	heat loss	SG
$W_{heat,PR}$	PR heating power	i	$c^L_{p,SG}$	liquid specific heat	SG
m_{SG}	SG mass flow rate	d	$c^V_{p,SG}$	vapor specific heat	SG
$T_{SG,SW}$	SG inlet water temperature	d	$c_{p,PR}$	liquid specific heat	PR
p_{SG}	SG steam pressure	o	$W_{loss,PR}$	heat loss	PR

Table 1: Measured variables and constant parameters of the model (Notations: **s**tate, **i**nput, **o**tput, **d**isturbance)

constraints prescribed for ν as follows: $v_{\min} \le v \le v_{\max} \Leftrightarrow N_{\min} p(v_{\min}) \le \nu \le N_{\min} p(v_{\max})$, where $p(v_{\min}) < 0 < p(v_{\max})$ and $N_{\min}$ is a physical limit for which $0 < N_{\min} \le N$ always holds.

Since $\frac{dT_{PC}}{dt}$ does not depend on m_{PR}, equation (6) can be substituted into equations (1)-(5) without producing algebraic loop. Carrying out this simple manipulation and centering the model around a predefined operating point $(\bar{N}, \bar{T}_{PC}, \bar{T}_{SG}, \bar{T}_{PR}, \bar{\nu}, \bar{W}_{heat,PR})$ the dynamic model can be rewritten in the following more compact form:

$$\begin{aligned} \dot{s} &= As + Bu_1 \\ \dot{z}\,|_{m_{PR}>0} &= (a_s^T + zp^T + s_2 q^T)s + a_z z + bu_2 \\ \dot{z}\,|_{m_{PR}\le 0} &= bu_2 \\ m_{PR} &= m_s^T s \end{aligned} \qquad (7)$$

where $s = [N - \bar{N}, T_{PC} - \bar{T}_{PC}, T_{SG} - \bar{T}_{SG}]$, $z = T_{PR} - \bar{T}_{PR}$,$u_1 = \nu - \bar{\nu}$, $u_2 = W_{heat,PR} - \bar{W}_{heat,PR}$ and $A, B, a_x, a_z, p, q, b, m_s$ are constant matrices, vectors of appropriate dimensions. If the state variables s_2 and z in the nonlinear terms are considered as time-varying parameters $\rho_1 = s_2, \rho_2 = z$ the equations above take the following hybrid-LPV form:

$$\begin{aligned} \dot{x}|_{m_{PR}>0} &= (A_{c,0} + \rho_1 A_{c,1} + \rho_2 A_{c,2})x + B_c u \\ &= A_c(\rho)x + B_c u \\ \dot{x}|_{m_{PR}\le 0} &= A_{c,3}x + B_c u \\ m_{PR} &= m^T x \end{aligned} \qquad (8)$$

where $A_{c,0}, A_{c,1}, A_{c,2}, A_{c,3}$ and B_c are constant matrices, and $x = \begin{bmatrix} s & z \end{bmatrix}^T$, $m = \begin{bmatrix} m_s^T & 0 \end{bmatrix}^T$.

2.3 Discrete time model

To apply model predictive control, the continuous model of the system has to be discretized. If $m_{PR} \le 0$ the dynamics is linear, so it can be easily transformed into discrete-time. We have to concentrate only on the first, parameter varying subsystem. Since the parameters ρ_1 and

ρ_2 vary relatively slowly in time, the discretization can be performed in the following way: at a time instant t_k the parameters are fixed and the linear system obtained is discretized by computing its solution under constant input u_k:

$$\begin{aligned} x(t_k + T_s) &\approx \\ x_{k+1} &= A_d(k)x_k + B_d(k)u_k \\ A_d(k) &= e^{A_c(\rho(t_k))T_s} \\ B_d(k) &= \int_0^{T_s} e^{A_c(\rho(t_k))(T_s-\tau)} B_c \, d\tau \end{aligned} \tag{9}$$

Fortunately, the computation of $A_d(k), B_d(k)$ can be simplified if the special structure of the matrices $A_{c,i}$ is exploited. By calculating the spectral decomposition of $A_c(\rho)$ symbolically (with parameters ρ_1, ρ_2) it can be seen that its eigenvalues are all distinct and do not depend on the parameters. Thus

$$e^{A_c(\rho(t_k))T_s} = V(\rho(t_k))\text{diag}(e^{\lambda_i})V(\rho(t_k))^{-1} \tag{10}$$

where $V(\rho) = V_0 + \rho_1 V_1 + \rho_2 V_2$. Continuing the analysis, we can see that the eigenvectors $V(\rho)$ are also of special form, which enables us to express the matrices $A_d(k), B_d(k)$ as follows:

$$\begin{aligned} A_d(k) &= A_{d,0} + \rho_1 A_{d,1} + \rho_2 A_{d,2} \\ A_{d,0} &= e^{A_{c,0}T_s} \\ A_{d,i} &= e^{(A_{c,0}+A_{c,i})T_s} - A_{d,0}, \; i = 1, 2 \\ B_d(k) &= B_{d,0} + \rho_1 B_{d,1} + \rho_2 B_{d,2} \\ B_{d,0} &= \int_0^{T_s} e^{A_{c,0}(T_s-\tau)} B_c \, d\tau \\ B_{d,i} &= \int_0^{T_s} e^{(A_{c,0}+A_{c,i})(T_s-\tau)} B_c \, d\tau - B_{d,0} \end{aligned} \tag{11}$$

Thus, the hybrid LPV form (8) is *preserved* after the discretization:

$$\begin{aligned} x_{k+1}|_{m_{PR}>0} &= (A_{d,0} + \rho_1 A_{d,1} + \rho_2 A_{d,2})x_k + \\ &\quad (B_{d,0} + \rho_1 B_{d,1} + \rho_2 B_{d,2})u_k \\ x_{k+1}|_{m_{PR}\le 0} &= A_{d,3}x_k + B_{d,3}u_k \\ m_{PR} &= m^T x_k \end{aligned} \tag{12}$$

The MPC framework applied later requires the system to be in *polytopic* form [5]. For this, we have to introduce upper and lower bounds for the parameters $\underline{\rho}_1 \le \rho_1 \le \overline{\rho}_1$, $\underline{\rho}_2 \le \rho_2 \le \overline{\rho}_2$, to be able to express the dynamics in the required form.

$$\begin{gathered} x_{k+1}^{m_{PR}>0} = A(k)x_k + B(k)u_k \quad [A(k), B(k)] \in \Omega \\ \Omega = \text{Co}\{[A_1, B_1], ..., [A_4, B_4]\} \\ A_i = A_{d,0} + \delta_{i,1} A_{d,1} + \delta_{i,2} A_{d,2} \\ B_i = B_{d,0} + \delta_{i,1} B_{d,1} + \delta_{i,2} B_{d,2} \\ \delta_{i,1} \in \{\underline{\rho}_1, \overline{\rho}_1\} \; \delta_{i,2} \in \{\underline{\rho}_2, \overline{\rho}_2\} \\ x_{k+1}|_{m_{PR}\le 0} = A_{d,3}x_k + B_{d,3}u_k \\ m_{PR} = m^T x_k \end{gathered} \tag{13}$$

Notice that, if we complete the set of corner points of Ω with the system $[A_5, B_5] = [A_{d,3}, B_{d,3}]$ the hybrid dynamics above can be *embedded* into the following *non-hybrid* LPV system:

$$\begin{aligned} x_{k+1} = A(k)x_k + B(k)u_k \quad [A(k), B(k)] \in \Omega \\ \Omega = \mathrm{Co}\{[A_1, B_1], ..., [A_4, B_4], [A_5, B_5]\} \end{aligned} \tag{14}$$

where $\mathrm{Co}(\cdot)$ denotes the convex hull of its arguments. This can be easily checked by considering the following convex combinations:

$$A(k) = \sum_{i=1}^{5} \gamma_1 A_i, \quad B(k) = \sum_{i=1}^{5} \gamma_i B_i, \quad \sum_{i=1}^{5} \gamma_i = 1 \tag{15}$$

with $\sum_{i=1}^{4} \gamma_i = 1,\ \gamma_5 = 0$ if $m_{PR} > 0$ and $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0,\ \gamma_5 = 1$ if $m_{PR} \leq 0$.

3 Controller Design

3.1 Control goals, assumptions and constraints

In the present control configuration, the neutronflux and the heating in the pressurizer are controlled separately. This performs quite well in the neighborhood of the prescribed steady states, but during large load changes, the temperature in the pressurizer usually slightly goes out of the required optimal operating interval. Therefore, the goal of the controller design is to obtain such a controller that - first of all - keeps all the predefined hard constraints for the state and input variables and secondly, it produces a satisfactorily quick load change transient. More precisely, the aim is to design an integrated controller, which steers the system from one operating point to another, so that

- the settling time of the neutron flux N be as small as possible
- the temperature change in the pressurizer be at most $1K$ during the transient
- the control inputs $\nu, W_{heat,PR}$ satisfy the given hard, physical constraints, coming from the limited heating energy at the pressurizer.

3.2 Model Predictive Control using Linear Matrix Inequalities

The control method we intend to apply is based on the MPC procedure proposed by Morari et.al in [5] and [2]. First, this procedure will be introduced briefly.

Suppose the system to be controlled is given in the form of (14) with an output equation $y(k) = Cx(k),\ y(k) \in \mathbb{R}^{n_y}$. Concentrating on the robust regulation problem, i.e. steering the state from an arbitrary initial value x_0 to the origin the MPC solution proposed by [5] involves the following min-max optimization problem:

$$\min_{u_{k+i|k}, i=0,1,...,m} \ \max_{[A(k+i), B(k+i)] \in \Omega} J_k^{\infty}, \qquad J_k^{\infty} = \sum_{i=0}^{\infty} \left(x_{k+i|k}^T Q_1 x_{k+i|k} + u_{k+i|k}^T R u_{k+i|k} \right) \tag{16}$$

where J_k^{∞} is a prescribed infinite horizon cost function and $x_{k+i|k}$, $u_{k+i|k}$ denote the predicted state and control action at time instant $k+i$, both based on the state measurement $x_k = x_{k|k}$.

This optimization has to be performed at each sampling instant with the actual state measurements to obtain the next control input. Since this problem is computationally demanding, the following idea has been applied: if it is possible to find a quadratic function $V(x_k) = x_k^T P_k x_k$, which gives an upper bound on the robust performance objective J_k^∞, the min-max problem can be replaced by a minimization of this quadratic function over the sequences of possible control moves. This can be easily solved by using linear matrix inequalities [9]. For $V(x_k)$ to be an upper bound it has to satisfy the following inequality [5]:

$$V(x_{k+i+1|k}) - V(x_{k+i|k}) \leq -\left(x_{k+i|k}^T Q_1 x_{k+i|k} + u_{k+i|k}^T R u_{k+i|k}\right) \tag{17}$$

Since in this case

$$-V(x_k) \leq -J_k^\infty \Rightarrow \max_{[A(k+i),B(k+i)]\in\Omega} J_k^\infty \leq V(x_k) \tag{18}$$

holds. Using this upper bound, (16) can be replaced by the following simpler problem:

$$\min_{u_{k+i|k}, i=0,1,\ldots,m} \max_{[A(k+i),B(k+i)]\in\Omega} J_k^\infty \leq \min_{u_{k+i|k}, i=0,1,\ldots} V(x_k) = \min_{u_{k+i|k}, i=0,1,\ldots} x_k^T P_k x_k \tag{19}$$

If $u_{k+i|k}$ is chosen to be $u_{k+i|k} = F_k x_{k+i|k}$ it can be shown (see [5]) that the solution (F_k, P_k) of (19) can be obtained in the following form:

$$F_k = F = YQ^{-1}, \;\; P_k = P = \gamma Q^{-1} \tag{20}$$

where $Q > 0, \gamma > 0, Y$ are the solutions of the following linear objective minimization problem:

$$\min_{\gamma,Q,Y} \gamma \tag{21}$$

subject to

$$\begin{bmatrix} 1 & x^T \\ x & Q \end{bmatrix} \geq 0 \tag{22}$$

and

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{\frac{1}{2}} & Y^T R^{\frac{1}{2}} \\ A_j Q + B_j Y & Q & 0 & 0 \\ Q_1^{\frac{1}{2}} Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \qquad j = 1..L \tag{23}$$

where $x = x_k$ and (23) is equivalent to the condition (17). If there are constraints prescribed for the input and/or output, they can be easily taken into consideration by expressing them as LMIs and attaching them to the constraints (22), (23) in the optimization problem above. For example, consider the peak bounds prescribed on each component of the input u_k, i.e.

$$|u_{j,k+i|k}| \leq u_{j,max}, \;\; j = 1..n_u \tag{24}$$

where n_u is the number of inputs. This holds if

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0, \quad X_{jj} \leq u_{j,max}^2 \tag{25}$$

The component-wise peak bounds on the outputs involve further LMI constraints, that can be given as:

$$\begin{bmatrix} Q & (A_jQ + B_jY)^T C_l^T \\ C_l(A_jQ + B_jY) & y_{l,max}^2 I \end{bmatrix} \geq 0$$
$$j = 1, ..., L, \quad l = 1...n_y \tag{26}$$

where C_l is the l-th row of C. For further details see [5] and [1]. The following Theorem summarizes the main result of [5]:

Theorem 1. *If at any time k there exists Q, Y, γ and $F_k = YQ^{-1}$ solving the problem* (21) *then*

- *the control input $F_k x_{k+i|k}$ will be feasible for all times $i > 0$*
- *the control policy given by the MPC procedure stabilizes the polytopic system and satisfies the prescribed input/output constraints.*

3.3 Implementation issues

The MPC procedure above assumes that the control gain F_k is available at the same time instant k, when the measurement x_k is taken. This means that the computation time of F_k (the time needed to solve the optimization problem above) is neglected, or assumed to be negligible compared to the sampling time T_s. Unfortunately, in our case this assumption does not hold. The reactor dynamics is sampled with $T_s = 1sec$, while the solution of the LMI-s (23),(25) and (26) takes minimum $0.7\,sec$ maximum $1.02\,sec$ (depending on the actual state measurement x_k). Decreasing the frequency of the controller update is not enough to solve this problem, since in itself it does not provide more time for computation. The following procedure is proposed instead: new controller is designed only at each M-th time instant; between two controller design steps the feedback gain is calculated as follows:

$$\tilde{F}_i = F_{k-M} \quad \text{if} \;\; i - k \leq l \tag{27}$$

$$\tilde{F}_i = F_{k-M} + \frac{i-k-l}{M-l}(F_k - F_{k-M}) \quad \text{if} \;\; i - k > l$$
$$i = k, ..., k + M - 1, \; k = n \cdot M, \; \forall n \tag{28}$$

After measuring x_k at the step k there are l time steps ($l \cdot T_s\, sec$) to determine the new control input F_k. During this time the system is controlled by the previous controller F_{k-M}. After having determined F_k we give it to the plant step by step, according to the interpolating rule above. The cause why F_k is not applied immediately is the observation that our system is sensitive to the change of the control gain. This means that small changes in F cause undesired oscillations in the system trajectories, especially in m_{PR}. The linear interpolation attenuates this effect.

The modified control strategy does not necessarily inherit the advantageous properties (constraint satisfaction, feasibility, stb.) of the original control policy. To ensure the feasibility of $\tilde{F}_i$ the following slightly conservative method has been chosen: at time k the gain F_k is designed so that the the controlled system is stable and satisfy the constraints *for all* convex combinations of the new and the previous control gains, i.e.:

$$\tilde{F} = \alpha F_{k-M} + (1-\alpha) F_k \tag{29}$$

Replacing the control input $u_k = F_k x_{k+i|k}$ with $u_k = \tilde{F} x_{k+i|k}$ and following the same argument as [5] the new LMI conditions can be easily recalculated. The LMIs (23),(25) and (26) have to be replaced by the LMIs (30), (31) and (32), respectively, which are defined as follows:

LMIs (23) and

$$\begin{bmatrix} Q & QA_j^T + Q\bar{F}^T B_j^T & QQ_1^{\frac{1}{2}} & Q\bar{F}^T R^{\frac{1}{2}} \\ A_j Q + B_j \bar{F}^T Q & Q & 0 & 0 \\ Q_1^{\frac{1}{2}} Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} \bar{F} Q & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \quad j = 1..L \tag{30}$$

LMIs (25) and

$$\begin{bmatrix} X & \bar{F}Q \\ Q\bar{F}^T & Q \end{bmatrix} \geq 0, \quad X_{jj} \leq u_{j,max}^2 \tag{31}$$

LMIs (26) and

$$\begin{bmatrix} Q & (A_j Q + B_j \bar{F} Q)^T C_l^T \\ C(A_j Q + B_j \bar{F} Q) & y_{l,max}^2 I \end{bmatrix} \geq 0 \tag{32}$$

where $F = F_k$, $P = P_k$, $\bar{F} = F_{k-M}$. It can be seen that the new sets of LMIs, beside the original F-dependent inequalities, contain further LMIs, which depend on the previous control gain $\bar{F}$. For further details see [8]. The modified control problem, therefore, can be handled in the same way as the original one, except that it involves more LMI constraints.

The properties of the modified control policy can be summarized in the following theorem.

Theorem 2. *(a) The control gain F_k defines a* feasible *control policy for all time $t > k$.*

(b) The control law obtained by using the modified MPC algorithm stabilizes *the closed loop system, and* satisfies the input and state constraints.

Proof. The proof is based on showing that the control policy

$$\begin{aligned} \tilde{F}_i &= F_{k-M} + \frac{i-k-l}{M-l}(F_k - F_{k-M}) \\ & \quad i = k \dots k+M-1 \\ \tilde{F}_i &= F_k \quad i \geq k+M \end{aligned} \tag{33}$$

(which is the control policy (28) extended to infinite horizon) is feasible at all time. The details of the proof can be found in [8]

Remark. Although (30), (31) and (32) contains more LMI-s than (23),(25) and (26), the number of decision variables is the same in the two optimization problem. Therefore the modified control policy does not require significantly more computation time than the original one.

Remark. Our algorithm requires high computational power only at the M-th time steps, when the LMI-s (30), (31) and (32) are solved. In other sampling instants a much simpler (low-level) computing hardware is enough to realize the interpolation (28). Therefore the modified algorithm can be implemented on a computer architecture depicted in figure 4. When there is no need for the high capacity computer, it can be used to solve other tasks related to the power plant.

4 Simulation Results

The control method was tested on an identified model of the pressurized water nuclear power plant. The dynamics was sampled with $T_s = 1s$ and it was centered around the operating point belonging to $\bar{N} = 100\%$ and $\bar{T}_{PR} = 599K$. The remaining two state variables were determined by substituting $\bar{N}, \bar{T}_{PR}$ into (1)-(5) and solving the equations for 0. The steady-state values obtained for T_{PC}, T_{SG} and the control inputs were as follows:

$$\begin{aligned} \bar{T}_{PC} &= 553.7398 \qquad \bar{T}_{SG} = 530.3435 \\ \bar{u}_1 &= 4.5312 \qquad \bar{u}_2 = 1.6823 \end{aligned} \tag{34}$$

In the simulation we examined the behavior of the plant under *load increase*, i.e. when the states are steered to the origin (to the steady state (34)) from a workpoint belonging to a lower neutron flux. In our case $\bar{N}(0) = 85\%$ and

$$\bar{T}_{PC}(0) = 548.9279 \qquad \bar{T}_{SG}(0) = 529.4117 \tag{35}$$

In the centered model these values are equivalent to the following initial state:

$$x(0) \quad = \quad -\begin{bmatrix} 15 & 4.8119 & 0.9318 & 0 \end{bmatrix}^T \tag{36}$$

Since we had constraints prescribed for N and T_{PR}, these two state variables were chosen as outputs, i.e.:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{37}$$

The constraints were as follows:

$$\begin{aligned} -20 \le x_1 \le 20 & \qquad (\bar{N} - 10 \le N \le \bar{N} + 10) \\ -1 \le x_4 \le 1 & \qquad (\bar{T}_{PR} - 1 \le T_{PR} \le \bar{T}_{PR} + 1) \end{aligned} \tag{38}$$

The control inputs always have to be in a physically realizable range. This means for $W_{heat,PR}$ to be between 0 and 3.6 and for ν to be between -15 and 15. Since in the MPC framework the limits have to be symmetric to 0 we used the following constraints:

$$\begin{aligned} -10 \le u_1 \le 10 & \\ -\bar{u}_2 \le u_2 \le \bar{u}_2 & \quad (0 \le W_{heat,PR} \le 3.3646) \end{aligned} \tag{39}$$

Figure 2: Trajectories of $N, T_{PC}, T_{SG}, T_{PR}$

The weighting matrices in the cost function were chosen to be $Q_1 = \mathrm{diag}(0.1, 0.1, 0.1, 0.01)$, $R = \mathrm{diag}(0.01, 0.01)$. The feedback gain was updated at every 12s ($M = 12$). For the computation 2s was allocated ($l = 2$).

The dynamic behavior of the system controlled by the modified MPC procedure can be seen in figs. 2 and 3. These figures show that the algorithm is able to solve the control problem: the settling time of the neutron flux is acceptably small, the states and the control inputs satisfy the prescribed constraints.

The simulation was performed in MATLAB/SIMULINK by using LMI Control Toolbox. The computation time of the control gain at each time step was between 0.91sec and 1.42sec on a P4 2.4 GHz processor. Since these values are smaller than the allocated time $l \cdot T_s = 2sec$ we can conclude that the modified control procedure is suitable for real-time application.

5 Conclusions

In this paper an LMI based model predictive regulator has been constructed for the primary circuit of a pressurized water nuclear power plant. During the control design it was shown that the dynamic behavior of the plant can be described well by a continuous hybrid LPV dynamics. Moreover, this model could be discretized so that the discrete time model obtained is also of LPV form with the same parameters as its continuous counterpart. Then, the discrete time hybrid model was embedded into a non-hybrid LPV structure, for which, effective control design methods exist. For the discrete LPV model we have successfully applied the LMI-based MPC algorithm proposed by [5]. Finally, a useful modification of the original control algorithm has been proposed to better suit it to our special needs. The dynamic behavior of the controlled system was investigated through numerical simulations and it has been found to satisfy the input and state constraints.

Further work will be directed towards two possible improvements of the proposed method. Firstly, the modeling of the real actuator dynamics of the neutronflux controller and secondly,

Figure 3: Control inputs u_1, u_2 and mass flow m_{PR}

Figure 4: Hardware architecture realizing the modified control policy

the treatment of some parameters (especially m_{SG} and $T_{SG,SW}$) as time-varying parameters in the LPV model.

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