Abstract—This paper presents the modeling and identification procedure for a VVER-type pressurized water reactor. The modeling goal is to produce a mathematical description in nonlinear state-space form that is suitable for control-oriented model analysis and preliminary controller design experiments. The proposed model takes temperature effects and Xenon poisoning into consideration and thus it is an extension of formerly published simpler model structures. Real transient measurement data from the plant has been used for the identification that is based on standard prediction error minimization. It is shown that the model is fairly well identifiable and the newly inserted model components significantly improve the quality of fit between the measured and computed model outputs. Furthermore, the estimated parameter values fall into physically meaningful ranges.

I. INTRODUCTION

The requirement for the continuous improvement of process safety and effectivity often necessitates the dynamic analysis and/or re-design of certain subsystems in complex plants. The need for continuous development is particularly true for such a safety-critical application like a nuclear power plant. For the dynamic analysis or controller synthesis, reliable dynamic models are needed where the level of detail and descriptive capability very much depend on the exact modeling goal [9].

Most modern controller design methods and the corresponding analysis techniques require that the mathematical model of the system is in the form of (a preferably low number of) ordinary differential equations [10]. Unfortunately, the traditionally available and commonly used dynamic models for nuclear power plants are much too complex and detailed for control purposes [8], [13].

Earlier, we developed different versions of a dynamic model for the primary circuit of a VVER-type nuclear power plants [5], [6], [7]. The domain of these former models included the dynamic behavior in normal operating mode together with the load changes between the day and night periods, which is approximately the 80 – 100% thermal power range. The reactor sub-model of our primary circuit model was a time-dependent, point kinetic model with a single type of delayed neutron emitting nuclei whose concentration was in a quasi steady-state [4], [11]. The effect of the control rod position on the reactivity was approximated by a quadratic function, i.e. the original simple reactor model was the following:

\[ \frac{dN}{dt} = \frac{p_1v^2 + p_2v + p_3}{\Lambda} N + S \]  

where \( N \) [%] is the neutron flux, \( v \) [m] is the control rod position, \( S \) [%/s] is a virtual neutron source and \( p_i, i = 1, 2, 3 \) are estimated parameters. This model suffers from the following shortcomings.

- The neutron flux is independent from the temperatures, i.e. it does not contain a temperature feedback from the temperature of the moderator and/or the fuel.
- To be able to reproduce steady-states correctly with only one differential equation, a virtual neutron source term \( S \) has been included. This is accepted and used in the literature, but in a more detailed model the introduction of other elements with clear physical meaning would be desirable.
- The concentrations of the delayed neutron emitting nuclei are assumed to be in quasi steady-state, that is far from being realistic.
- The model does not describe the measured trends in the neutron flux in the neighborhood of the steady states and it has some inaccuracies when simulating load increase following a load decrease transient (see, Fig. 2).

Process knowledge suggests that taking into consideration some additional physical details, a more accurate model could be obtained with a bit more but still manageable number of state variables. Therefore, the aim of this paper is to extend the simple reactor model in Eq. (1) with temperature feedbacks, the dynamics of delayed neutron emitting nuclei and xenon poisoning. Beside the new model structure, the parameter estimation procedure is also presented in this paper using measured plant data.

The paper is organized as follows. In the second section, the extended model in state space form is presented. The third section describes the parameter estimation method and the measurements. The fourth section contains the results of the parameter estimation, while the conclusions can be found in the fifth section.

II. REACTOR MODEL

A. Modeling Assumptions

In order to have a low order dynamic model of the reactor the following simplification assumptions have been made.

R1 The reactor is considered as a spatially homogeneous lumped parameter system. Therefore, the reactor model is a time-dependent, non-linear single-group model [11].
The dynamic model of the reactor is derived from the point kinetic equations.

Only a single "average" group of the delayed neutron emitting nuclei is assumed.

The reactor is composed of the fuel, the moderator and the control rod as modelling elements (balance volumes).

The reactivity dependence on the rod position is assumed to be quadratic.

The reactivity dependence on the temperatures is assumed to be linear.

The reactivity dependence on the xenon concentration is assumed to be linear.

The boron concentration is regarded to be constant during the simulation together with the reactivity coefficients.

The mass flow rate of the moderator is assumed to be constant.

The heat loss of the reactor is neglected.

The heat capacity of the moderator is assumed to be constant.

The reactivity and the xenon concentration are assumed to be linear.

According to the assumptions R1, R2 and R3, the neutron dynamics and the delayed neutron emitting nuclei dynamics are described by the following point kinetic equations [11]:

\[
\frac{dN}{dt} = \beta \frac{N}{X} (\rho - 1) + n_C \frac{\beta}{X}
\]

\[
\frac{d\lambda_C}{dt} = \lambda_C (N - n_C)
\]  

The reactivity depends on the temperatures, the control rod position and the xenon concentration (assumptions R5, R6 and R7) [3]:

\[
\rho = \alpha_f (T_f - T_{f0}) + \alpha_m (T_m - T_{m0}) + p_2 z^2 + p_1 z + p_0 + \frac{\sigma_X}{\beta z_f} (n_X - n_{X0})
\]

where \(\alpha_f (T_f - T_{f0})\) describes the temperature feedback of the fuel, \(\alpha_m (T_m - T_{m0})\) describes the temperature feedback of the moderator, \(p_2 z^2 + p_1 z + p_0\) is the effect of the rod to the reactivity and \(\frac{\sigma_X}{\beta z_f} (n_X - n_{X0})\) is the effect of the Xenon.

**Table I: Variables and Parameters**

<table>
<thead>
<tr>
<th>Identifier</th>
<th>M. u.</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{n}(t))</td>
<td>m</td>
<td>Rod position</td>
</tr>
<tr>
<td>(N(t))</td>
<td>%</td>
<td>Neutron concentration</td>
</tr>
<tr>
<td>(n_C(t))</td>
<td>%</td>
<td>Concentration of the delayed neutron emitting nuclei</td>
</tr>
<tr>
<td>(n_f(t))</td>
<td>cm(^{-3})</td>
<td>Iodine concentration</td>
</tr>
<tr>
<td>(n_X(t))</td>
<td>cm(^{-3})</td>
<td>Xenon concentration</td>
</tr>
<tr>
<td>(\rho(t))</td>
<td>$</td>
<td></td>
</tr>
<tr>
<td>(T_f(t))</td>
<td>°C</td>
<td>Temperature of the fuel</td>
</tr>
<tr>
<td>(T_m(t))</td>
<td>°C</td>
<td>Average temperature of the moderator</td>
</tr>
<tr>
<td>(T_{out}(t))</td>
<td>°C</td>
<td>Temperature of the water leaving the reactor</td>
</tr>
<tr>
<td>(T_{in}(t))</td>
<td>°C</td>
<td>Temperature of the water entering the reactor</td>
</tr>
<tr>
<td>(\phi_0)</td>
<td>cm(^{-3})s(^{-1})</td>
<td>Initial equilibrium neutron flux</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>s</td>
<td>Average generation time</td>
</tr>
<tr>
<td>(\Sigma_f)</td>
<td>cm(^{-1})</td>
<td>Macroscopic fission cross section</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-</td>
<td>Fraction of delayed neutron group</td>
</tr>
<tr>
<td>(\lambda_C)</td>
<td>s(^{-1})</td>
<td>Decay constant of the delayed neutron emitting nuclei</td>
</tr>
<tr>
<td>(\lambda_f)</td>
<td>s(^{-1})</td>
<td>Decay constant of Iodine</td>
</tr>
<tr>
<td>(\lambda_X)</td>
<td>s(^{-1})</td>
<td>Decay constant of Xenon</td>
</tr>
<tr>
<td>(\sigma_X)</td>
<td>cm(^2)</td>
<td>Microscopic absorption cross section</td>
</tr>
<tr>
<td>(Y_f)</td>
<td>-</td>
<td>Iodine yield</td>
</tr>
<tr>
<td>(Y_X)</td>
<td>-</td>
<td>Xenon yield</td>
</tr>
<tr>
<td>(\alpha_f)</td>
<td>$/^\circ C$</td>
<td>Temperature coefficient of the fuel</td>
</tr>
<tr>
<td>(\alpha_m)</td>
<td>$/^\circ C$</td>
<td>Temperature coefficient of the moderator</td>
</tr>
<tr>
<td>(A)</td>
<td>m(^2)</td>
<td>Area of a fuel rod</td>
</tr>
<tr>
<td>(U)</td>
<td>J m(^{-2}) K(^{-1})</td>
<td>Heat transfer coefficient between the fuel and the moderator</td>
</tr>
<tr>
<td>(M_f c_p)</td>
<td>J/K</td>
<td>Heat capacity of the fuel</td>
</tr>
<tr>
<td>(M_m c_p)</td>
<td>J/K</td>
<td>Heat capacity of the moderator</td>
</tr>
<tr>
<td>(m_p)</td>
<td>kg/s</td>
<td>Mass flow rate of the moderator</td>
</tr>
<tr>
<td>(F)</td>
<td>J/%</td>
<td>Reactor heat power per 1% of neutron flux</td>
</tr>
<tr>
<td>(p_0, p_1, p_2)</td>
<td>$\frac{1}{m}, \frac{1}{m}, m$</td>
<td>Rod parameters</td>
</tr>
</tbody>
</table>
2) Equations of Thermodynamics: Energy balances are constructed for the fuel and the moderator (assumption R4). The energy balance for the fuel is

\[ M_f c_p f dT_f = -UA(T_f - T_m)dt + FNpdt \]  

(5)

where \( M_f c_p f dT_f \) is the inner energy change of the fuel due to the temperature change, \(-UA(T_f - T_m)dt\) is the transferred heat to the moderator and \(FNpdt\) is the heat power of the reactor per unit time. To describe the temperature of the fuel Eq. (5) is transformed to the

\[ \frac{dT_f}{dt} = -\frac{UA}{M_f c_p f}(T_f - T_m) + \frac{F}{M_f c_p f}N \]  

(6)

form.

The energy balance for the moderator is

\[ M_m c_p m dT_m = UA(T_f - T_m)dt + m_p c_p m T_{in}dt - m_p c_p m T_{out}dt \]  

(7)

where \( M_m c_p m dT_m \) is the inner energy change of the moderator due to the temperature change, \( m_p c_p m T_{in}dt \) is the energy of the inlet mass flow of the moderator and \( m_p c_p m T_{out}dt \) is the energy of the outlet mass flow of the moderator (assumption R9). Applying a similar transformation as before we obtain that

\[ \frac{dT_m}{dt} = \frac{UA}{M_m c_p m}(T_f - T_m) - \frac{m_p}{M_m}(T_{out} - T_{in}) \]  

(8)

Let us group the parameters and introduce the following notations

\[ A_1 = \frac{UA}{M_f c_p f}, \quad A_2 = \frac{F}{M_f c_p f}, \quad A_3 = \frac{UA}{M_m c_p m}, \quad A_4 = \frac{m_p}{M_m} \]  

(9)

With this we can transform (6) and (8) into the following form

\[ \frac{dT_f}{dt} = -A_1(T_f - T_m) + A_2 N \]  

(11)

\[ \frac{dT_m}{dt} = A_3(T_f - T_m) - A_4(T_{out} - T_{in}) \]  

(12)

However, the steady state determines the following relationships among these parameters:

\[ 0 = -A_1(T_{f0} - T_{m0}) + A_2 N_0 \]  

(13)

\[ 0 = A_3(T_{f0} - T_{m0}) - A_4(T_{out0} - T_{in0}) \]  

From these we can express \( A_2 \) and \( A_4 \) as:

\[ A_2 = \frac{A_1(T_{f0} - T_{m0})}{N_0} \]  

(14)

\[ A_4 = \frac{A_3(T_{f0} - T_{m0})}{(T_{out0} - T_{in0})} \]  

(15)

Furthermore, the expression \( T_{out0} - T_{in0} \) in the denominator of \( A_4 \) can be transformed into \( 2(T_{m0} - T_{in0}) \) using the equation \( T_{m0} = T_{out0} + T_{in0} \).

3) Equations of Poisening: In power reactors we cannot neglect the xenon poisoning, because the duration of load changes (4-6 hours) between the day and night is close to the half life times of the Xenon (9 hours) and the Iodine (6 hours). The concentration change of Xenon is also relevant in shorter terms, because it has great microscopic absorption cross section. To describe the concentration change of Xenon we also have to describe the Iodine concentration [4], [11]:

\[ \frac{dn_I}{dt} = Y_I \Sigma_f N_0 \phi_0 - \lambda_I n_I \]  

(16)

\[ \frac{dn_X}{dt} = Y_X \Sigma_f N_0 \phi_0 + \lambda_I n_I - \lambda_X n_X - \sigma_X n_X N \]  

(17)

where \( Y_I \Sigma_f N_0 \phi_0 \) and \( Y_X \Sigma_f N_0 \phi_0 \) is the effect of the Uranium fission, \( \lambda_I n_I \) is the Iodine atoms decay to xenon in one cm³ per time unit, \(-\lambda_X n_X - \sigma_X n_X N \) is the decrease of the xenon concentration because of the decay and neutron absorption.

We use the following simplifications in the notations: \( \lambda = n_X / \Sigma_f \) and \( I = n_I / \Sigma_f \).

C. State Space Form

In order to be able to apply standard identification and controller design methods, the above engineering model equations have been transformed into a state-space model form.

State equations

\[ \frac{dN}{dt} = \frac{dI}{dt} = \alpha N + 2(\alpha z + \beta + \lambda N_0) + \alpha (N - N_0) \]  

(18)

\[ \frac{dc}{dt} = -\lambda c + \lambda I \pm I \]  

(19)

\[ \frac{dX}{dt} = \lambda X - \lambda I - \sigma X N \]  

(20)

Output equation

\[ y = [N, T_m]^T \]  

(21)

III. PARAMETER IDENTIFICATION

A. Measurements

Measured data from unit 1 of the Paks Nuclear Power Plant in Hungary were collected for parameter estimation purposes. To extract as much dynamic information as possible, load change periods were selected for identification.

The measured data that are needed for the identification included the neutron flux \( N \), and the average temperature of the moderator \( T_m \) as outputs and the rod position \( z \) as input. The data source was the Verona system (see [2]) that is a reactor monitoring system storing also reactor data. Stored values are uniformly sampled, the sampling time is 10 s.

The time-span of the raw measurements was between 2 and 72 hours. The selected data sequences had to contain steady
state values together with power increase/decrease without any significant disturbances and operating mode changes. After the investigation of the measured data a time interval of 9.5 hours was chosen for parameter estimation.

It is important to note that these data are passive measured data from the nuclear power plant under closed-loop control where the excitation was provided by the load changes.

B. Identification Method

First, the parameters of the model have been grouped based on the knowledge of their values.

- **Known parameters.** They are the parameters of nuclear processes, Λ, β, λ₁, λ₂, Y₁, Y₂, and λ_C.

- **Partially known parameters.** A reliability domain is given to their value, they are φ₀, σ_X, α_m, α_f, and the rod parameters. The value of these parameters is in principle known from the literature, but because of the model simplification, the estimated value can be different from the literature value.

- **Unknown parameters:** A₁, A₃. We can only give an initial estimate of them from the geometry and thermohydraulic data of the reactor.

The value of partially known and unknown parameters have been estimated from measurements using a constraint during the estimation: the estimated value of partially known parameters must be in their reliability (physically meaningful) domain.

The above parameter estimation problem is basically an optimization problem with objective function \( f_{\text{obj}} \) which is bound constrained to keep some estimated parameter values in a physically meaningful range. Because of the existence of these constraints, the classical least squares (LS) method cannot be applied for identification. For the evaluation of \( f_{\text{obj}} \), the simulation of the system dynamics with some parameter vector \( \theta \) is required which is a computationally expensive operation. This means that the numerical approximation and evaluation of the gradient of \( f_{\text{obj}} \) requires much computational effort and moreover, it can often be unreliable because of the noise of some measurements. These facts motivated us to choose a simple yet effective numerical optimization method that does not need the computation of the gradient of the objective function.

The Parameter Estimation Tool of the Matlab is applied to implement the identification algorithm. The applied identification method is the Pattern search/Nelder-Mead search method. It is an optimization-based parameter estimation method based on the Nelder-Mead simplex method [12]. The objective function is the SSE (sum of square of error) which measures the data fit in terms of the 2-norm between the measured and the model-computed output signals.

The brief operational principle of the Nelder-Mead algorithm is the following (for details, see [12]): A simplex is the convex hull of \( n + 1 \) vertices in an \( n \)-dimensional space. The method starts from an initial working simplex which is created using the given initial parameter value. The algorithm then performs a sequence of transformations (that can be reflection, expansion, contraction or shrink) of the working simplex, to decrease the objective function values at the vertices. The algorithm is terminated when the size of the simplex is sufficiently small, or when the function values at the vertices are close to each other in some norm. In each iteration step, the algorithm typically needs only one or two objective function evaluations which is quite low compared to most other methods.

It is important to note that the simplex search algorithm (similarly to many nonlinear optimization techniques) does not guarantee that the obtained point is a global minimum on the whole parameter domain. Therefore it is very important to use as much prior information about the modeled process as possible to choose proper initial parameter values for the method.

To use the simplex method, suitable initial values are needed. They are given from [1] and from the discussion with nuclear power plant experts.

IV. RESULTS

The parameter estimation was based on the state-space model equations (15)-(20). The input variable to the model was the rod position \( z(t) \) and the input temperature \( T_{in}(t) \), the model output variables were the neutron flux \( N(t) \) and the moderator temperature \( T_m(t) \). The values of the parameters Λ, β, λ₁, λ₂, Y₁, Y₂, and λ_C were assumed to be known from the literature, while \( A_1, A_3, \alpha_f, \alpha_m, \phi_0 \) and \( \sigma_X \) were estimated using Nelder-Mead simplex algorithm. The applied measured signals can be seen in Fig. 1. We have to note that the rod position is measured as the difference between the nominal position of the rod and the current rod position. If the rod is inserted from there, then the rod position becomes positive.

The estimated parameter values can be seen in Table II, while the neutron flux and the temperature of the moderator fitting can be seen in Fig. 2 and Fig. 3, respectively.

The quality of the parameter estimation is investigated by the analysis of normalized error function as a function of each parameter, and by the analysis of the normalized error function as a function of some pairs of parameters. The normalized error function is defined as the SSE divided by the 2-norm of the measured signal. Some typical results can be seen in Figs. 4 and 5. The circle shows the estimated values in Fig. 4.

A. Discussion

The analysis of the fit: In Fig. 2 one can see the measured neutron flux, together with the simulated one of the original reactor model [5] and the neutron flux simulated by our new model.

The steady states are reproduced much better by the new model than by the original one. The dynamics of the measurements are also described better by our new model than the original one. This is particularly visible at the end of the power increase, where the original model shows a high overshoot while the new model fits well. In addition, the time constants of both the new and the original model correspond well to the dynamics of the real system shown by the measurements.
TABLE II

VALUES OF ESTIMATED PARAMETERS AND KNOWN CONSTANTS

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>7.5 - $10^{-12}$</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>2.85 - $10^{-18}$</td>
</tr>
<tr>
<td>$\alpha_f$</td>
<td>$-4.436 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>$-2.317 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.141</td>
</tr>
<tr>
<td>$p_0$</td>
<td>7.519 - $10^{-3}$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$-0.37811$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$-0.73374$</td>
</tr>
<tr>
<td>$Y_f$</td>
<td>0.0639</td>
</tr>
<tr>
<td>$Y_X$</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$2.6 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.0065</td>
</tr>
<tr>
<td>$\lambda_C$</td>
<td>7.728 - $10^{-2}$</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>2.9306 - $10^{-5}$</td>
</tr>
<tr>
<td>$\lambda_X$</td>
<td>2.1066 - $10^{-5}$</td>
</tr>
</tbody>
</table>

However, at the end of the measured signal time-span the neutron flux is increasing slowly as seen in Fig. 2. Unfortunately, none of our models can reproduce this slow change.

We have to note that application of six groups of delayed neutron emitting nuclei in the original model did not give better result, therefore we decided to apply only a single group of average delayed neutron emitting nuclei.

The analysis of the estimated values: The estimated values are acceptable, they are in their physically meaningful reliability domains.

- $p_0$, $p_1$, $p_2$ (Fig. 4 a.).
  - The error value as a function of the parameter is similar to a quadratic function, therefore a unique minimum exists. One can see that the estimated parameter values are close to the minimum value.
- $A_1$, $A_3$ (Fig. 4 b.).
  - The error value function as a function of the parameter is close to a constant. It contains a lot of local minima but their values are similar. This means that these parameters cannot be estimated properly.
- $\alpha_f$, $\alpha_m$ and $\sigma_X$ (Fig. 4 c. and d.).
  - The error value function is asymmetric. On one side its gradient is high, while in the other side it is close to zero. A unique minimum exists, but its determination can be problematic.

Based on the analysis of the error function as a function of two parameters (see Fig. 5) one can see that the estimation of some variable pairs are strongly correlated. For example, one can see from Fig. 5 a, that there is unique minimum of the error function as a function of $p_1$ and $p_2$ but there is a linear correlation between these two parameters, i.e. they cannot be
estimated totally independently. Similar, but stronger linear correlation exists between parameters $p_2$ and $\alpha_m$, see Fig. 5 b, therefore, only one of them can be estimated and this value determines the value of the other parameter.

V. CONCLUSION

A new extended reactor model for the control oriented modeling of the primary circuit of a nuclear power plant has been presented in this paper. In the new reactor model, the reactivity depends on the control rod position, the average temperature of the moderator, the temperature of the fuel, and on the poison processes. The introduction of previously unmodeled effects resulted in the fact that this more detailed model describes the system dynamics more precisely than before, and a good fit has been achieved even for the load changing transients. The model is not suitable (and not intended) for describing dynamics under non-standard operating conditions, such as faults.

The model parameters have been classified appropriately, and the partially known and the unknown model parameters have been estimated using a quadratic objective function and a nonlinear optimization algorithm, namely, the Nelder-Mead simplex search method. The necessary measurement data were collected from a unit of the Paks Nuclear Power Plant, located in Hungary. The quality of estimates has also been investigated by the analysis of the objective function.

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