Parameter Sensitivity Analysis of an Industrial Synchronous Generator

Attila Fodor, Attila Magyar, Katalin M. Hangos

***Abstract*— A previously developed simple dynamic model of an industrial size synchronous generator operating in a nuclear power plant is analyzed in this paper. The constructed state-space model consists of a nonlinear state equation and a bilinear output equation. It has been shown that the model is locally asymptotically stable with parameters obtained from the literature for a similar generator.**

The effect of load disturbances on the partially controlled generator has been analyzed by simulation using a PI controller. It has been found that the controlled system is stable and can follow the set-point changes in the effective power well. The sensitivity of the model for its parameters has also been investigated and parameter groups have been defined according to the system's degree of sensitivity to them. This groups form the different candidates of parameters for a subsequent parameter estimation.

I. INTRODUCTION

Nuclear power plants generate electrical power from nuclear energy, where the final stage of the power production includes a synchronous generator that is driven by a turbine. Similarly to other power plants, both the effective and reactive components of the generated power depend on the need of the consumers and on their own operability criteria. This consumer generated time-varying load is the major disturbance that should be taken care of by the generator controller.

The turbo generator, the subject of our study, is a specific synchronous generator with a special cooling system. The armature has been cooled by water and the rotor has been cooled by hydrogen. In the examined nuclear power station the exciter field regulator of the synchronous generator currently does not control the reactive power, only the effective power. *The final aim of our study is to design a controller that can control the reactive power such that its generation is minimized in such a way that the quality of the control of the effective power remains (nearly) unchanged.*

Because of the specialities and great practical importance of the synchronous generators in power plants, their modeling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1] and [2]), there are papers that describe the modeling and use the developed models for the design of various controllers [3], [4]. These papers, however, do not take the special circumstances found in nuclear power plants into account that may result in special generator models.

The aim of this paper is to perform model verification and parameter sensitivity analysis of a simple dynamic model of a synchronous generator proposed in [5]. The result of this analysis will be the basis of a subsequent parameter estimation step.

II. THE MODEL OF THE SYNCHRONOUS GENERATOR

In this section the bilinear state-space model for a synchronous generator is presented [5] that will be used for local stability analysis and parameter sensitivity analysis.

A. Modeling assumptions

For constructing the synchronous generator model, let us make the following assumptions:

- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- there are two amortisseur or damper windings in the machine,
- all of the windings are magnetically coupled,
- the flux linkage of the windings is a function of the rotor position,
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal,
- stator and rotor permeability are assumed to be infinite.

It is also assumed that all the losses due to wiring, saturation, and slots can be neglected.

The six windings (three stators, one rotor and two damper) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linking of the windings is also a function of the rotor position. The actual terminal voltage v of the windings can be written in the form

$$v = \pm \sum_{j=1}^{J} (r_j \cdot i_j) \pm \sum_{j=1}^{J} (\dot{\lambda}_j),$$

where i_j are the currents, r_j are the winding resistances, and λ_j are the flux linkages. The positive directions of the stator currents point out of the synchronous generator terminals.

Thereafter, the two stator electromagnetic fields, both traveling at rotor speed, were identified by decomposing each

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stator phase current under steady state into two components, one in phase with the electromagnetic field and an other phase shifted by 90^o. With the aboves, one can construct an airgap field with its maxima aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles) (see Fig. 1).

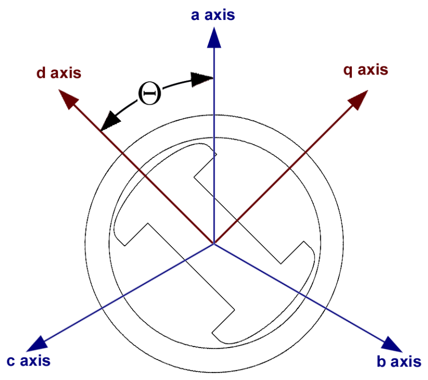

Fig. 1. The abc and $0dq$ frames of the generator

This method is called the Park's transformation that gives the following relationship:

$$\begin{aligned} \mathbf{i_{0dq}} &= \mathbf{P} \cdot \mathbf{i_{abc}} \\ \mathbf{i_{abc}} &= \mathbf{P^{-1}} \cdot \mathbf{i_{0dq}} \end{aligned} \quad (1)$$

where the current vectors are $\mathbf{i_{0dq}} = \begin{bmatrix} i_0 & i_d & i_q \end{bmatrix}^{\mathbf{T}}$ and $\mathbf{i_{abc}} = \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}^{\mathbf{T}}$ and the Park's transformation matrix is:

$$\mathbf{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ i_a \cos(\Theta) & i_b \cos(\Theta - \frac{2\pi}{3}) & i_c \cos(\Theta - \frac{4\pi}{3}) \\ i_a \sin(\Theta) & i_b \sin(\Theta - \frac{2\pi}{3}) & i_c \sin(\Theta - \frac{4\pi}{3}) \end{bmatrix}$$

where i_a, i_b and i_c are the phase currents and Θ [rad] is the angle between the phase current i_a and the current i_d. Park's transformation uses three variables: d and q axis components (i_d and i_q) and stationary current component (i_0), which is proportional to the zero-sequence current.

All flux components correspond to an electromagnetic field (EMF), the generator EMF is primarily along the rotor q axis. The angle between this EMF and the output voltage is the machine torque angle δ, where the phase a is the reference voltage of the output voltage. The position of the d axis (in radian) is $\Theta = \omega_r t + \delta + \pi/2$, where ω_r is the rated synchronous angular frequency. Finally, the the voltage and linkage equations are $\mathbf{v_{0dq}} = \mathbf{P} \cdot \mathbf{v_{abc}}$ and $\lambda_{\mathbf{0dq}} = \mathbf{P} \cdot \lambda_{\mathbf{abc}}$, where the vectors are $\mathbf{v_{0dq}} = \begin{bmatrix} v_0 & v_d & v_q \end{bmatrix}^{\mathbf{T}}$ and $\mathbf{v_{abc}} = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix}^{\mathbf{T}}$, and the linkage flux vectors are $\lambda_{\mathbf{0dq}} = \begin{bmatrix} \lambda_0 & \lambda_d & \lambda_q \end{bmatrix}^{\mathbf{T}}$ and $\lambda_{\mathbf{abc}} = \begin{bmatrix} \lambda_a & \lambda_b & \lambda_c \end{bmatrix}^{\mathbf{T}}$.

The value of the active power can be written (using (1)) in both coordinate systems:

$$p = \mathbf{v_{abc}^T i_{abc}} = \mathbf{v_{0dq}^T P P^{-1} i_{0dq}} = \mathbf{v_{0dq}^T i_{0dq}} \quad (2)$$

B. Flux linkage equations

The generator consists of six coupled coils referred to with indices a, b, c (the stator phases coils), F, D, and Q (the field coil, the d-axis amortisseur and the q-axis amortisseur). The linkage equations are in the following form:

$$\begin{bmatrix} \lambda_a & \lambda_b & \lambda_c & \lambda_F & \lambda_D & \lambda_Q \end{bmatrix}^T = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (3)$$

where L_{xy} is the coupling inductance of the coils. It is important to note that the inductances are time varying since Θ is a function of time. The time varying inductances can be simplified by referring all quantities to a rotor frame of reference through Park's transformation:

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \lambda_{\mathbf{abc}} \\ \lambda_{\mathbf{FDQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \cdot \\ \cdot \begin{bmatrix} \mathbf{L_{aa}} & \mathbf{L_{aR}} \\ \mathbf{L_{Ra}} & \mathbf{L_{RR}} \end{bmatrix} \begin{bmatrix} \mathbf{P^{-1}} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \mathbf{i_{abc}} \\ \mathbf{i_{FDQ}} \end{bmatrix}, \quad (4)$$

where $\mathbf{L_{RR}}$ is the rotor-rotor, $\mathbf{L_{aa}}$ is the stator-stator, $\mathbf{L_{aR}}$ and $\mathbf{L_{Ra}}$ are the stator-rotor inductance matrices. $\mathbf{P}$ is the Park's transformation matrix, $\mathbf{I_3}$ is the 3×3 unit matrix. The obtained transformed flux linkage equations are as follows:

$$\begin{bmatrix} \lambda_0 & \lambda_d & \lambda_q & \lambda_F & \lambda_D & \lambda_Q \end{bmatrix}^T = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (5)$$

where:

$$\begin{aligned} L_d &= L_s + M_s + \tfrac{3}{2} L_m & L_q &= L_s + M_s - \tfrac{3}{2} L_m \\ L_0 &= L_s - 2M_s & k &= \sqrt{\tfrac{2}{3}} \end{aligned} \quad (6)$$

C. Voltage equations

The schematic equivalent circuit of the synchronous machine can be seen in Fig. 2, and the voltage equations (7) can be derived from it.

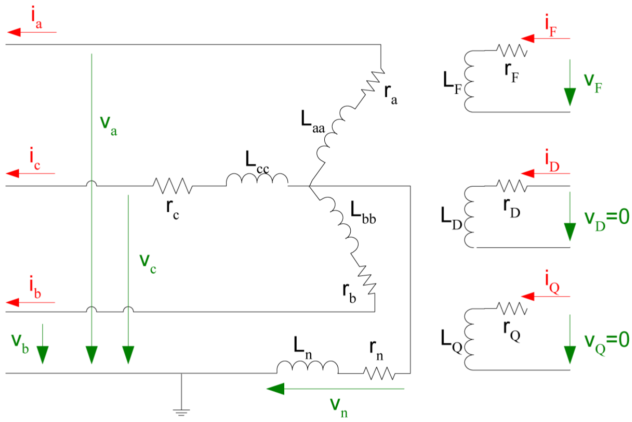

Fig. 2. The simplified schema of the synchronous machine

$$\begin{bmatrix} \mathbf{v_{abc}} \\ \mathbf{v_{FDQ}} \end{bmatrix} = -\begin{bmatrix} \mathbf{R_{abc}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix} \begin{bmatrix} \mathbf{i_{abc}} \\ \mathbf{i_{FDQ}} \end{bmatrix} - \\ - \begin{bmatrix} \dot{\lambda}_{\mathbf{abc}} \\ \dot{\lambda}_{\mathbf{FDQ}} \end{bmatrix} + \begin{bmatrix} \mathbf{v_n} \\ 0 \end{bmatrix}, \quad (7)$$

where $\mathbf{R_{abc}} = \text{diag}(\begin{bmatrix} r_a & r_b & r_c \end{bmatrix})$, and $\mathbf{R_{FDQ}} = \text{diag}(\begin{bmatrix} r_F & r_D & r_Q \end{bmatrix})$.

The neutral voltage $\mathbf{v_n}$ can also be deduced from Fig. 2 as follows:

$$\mathbf{v_n} = -\mathbf{R_n i_{abc}} - \mathbf{L_{nm}} \dot{\mathbf{i}}_{\mathbf{abc}}, \tag{8}$$

where $\mathbf{L_{nm}} = L_n \mathbf{U_3}$, and $\mathbf{R_n} = r_n \mathbf{U_3}$, and $\mathbf{U_3}$ denotes the 3×3 matrix of full ones.

The direct, quadratic, field and amortisseur component of the voltage using Park's transformation:

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \mathbf{v_{abc}} \\ \mathbf{v_{FDQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{v_{0dq}} \\ \mathbf{v_{FDQ}} \end{bmatrix} \tag{9}$$

Using (1), it is possible to expand the voltages of the resistances from (9) as

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \mathbf{R_{abc}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix} \begin{bmatrix} \mathbf{i_{abc}} \\ \mathbf{i_{FDQ}} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{P R_{abc} P^{-1}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix} \begin{bmatrix} \mathbf{i_{0dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{\hat{R}_{abc}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix} \begin{bmatrix} \mathbf{i_{0dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix}.$$

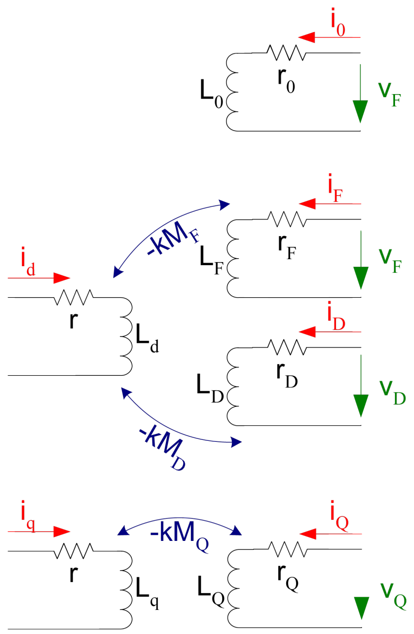

Fig. 3. The simplified equivalent circuit of the transformed stator and rotor circuits

Using the initial assumption of symmetrical tri-phase stator windings (i.e. $r_a = r_b = r_c = r$), we obtain $\mathbf{\hat{R}_{abc}} = \mathbf{R_{abc}} = \text{diag}(\begin{bmatrix} r & r & r \end{bmatrix})$.

The time derivatives of the fluxes can be computed similarly

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \dot{\lambda}_{\mathbf{abc}} \\ \dot{\lambda}_{\mathbf{FDQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{P}\dot{\lambda}_{\mathbf{abc}} \\ \dot{\lambda}_{\mathbf{FDQ}} \end{bmatrix}, \tag{10}$$

where

$$\mathbf{P}\dot{\lambda}_{\mathbf{abc}} = \dot{\lambda}_{\mathbf{0dq}} - \dot{\mathbf{P}}\lambda_{\mathbf{abc}} = \dot{\lambda}_{\mathbf{0dq}} - \dot{\mathbf{P}}\mathbf{P}^{-1}\lambda_{\mathbf{0dq}},$$

and the last term is

$$\dot{\mathbf{P}}\mathbf{P}^{-1}\lambda_{\mathbf{0dq}} = \omega \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \end{bmatrix} = \begin{bmatrix} 0 \\ -\omega\lambda_q \\ \omega\lambda_d \end{bmatrix}$$

Finally, the neutral voltage is derived as

$$\begin{bmatrix} \mathbf{v_{0dq}} \\ \mathbf{v_{FDQ}} \end{bmatrix} = - \begin{bmatrix} \mathbf{R_{0dq}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix} \begin{bmatrix} \mathbf{i_{0dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix}$$

$$- \begin{bmatrix} \dot{\lambda}_{\mathbf{0dq}} \\ \dot{\lambda}_{\mathbf{FDQ}} \end{bmatrix} + \begin{bmatrix} \dot{\mathbf{P}}\mathbf{P}^{-1}\lambda_{\mathbf{0dq}} \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{n_{0dq}} \\ 0 \end{bmatrix} \tag{11}$$

where $\mathbf{n_{0dq}}$ is the voltage drop from the neutral network.

$$\mathbf{n_{0dq}} = \mathbf{P v_n} = -\mathbf{P R_n P^{-1} P i_{abc}} - \mathbf{P L_{nm} P^{-1} P} \dot{\mathbf{i}}_{\mathbf{abc}} =$$

$$-\mathbf{P R_n P^{-1} i_{0dq}} - \mathbf{P L_{nm} P^{-1}} \dot{\mathbf{i}}_{\mathbf{0dq}} = \begin{bmatrix} -3r_n i_0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -3L_n \dot{i}_0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

In balanced condition the v_0 voltage is 0. The above equation can be written in the following form:

$$\begin{bmatrix} \mathbf{v_{dq}} & \mathbf{v_{FDQ}} \end{bmatrix}^T =$$

$$- \begin{bmatrix} R & 0 \\ 0 & R_R \end{bmatrix} \begin{bmatrix} \mathbf{i_{dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_{\mathbf{dq}} \\ \dot{\lambda}_{\mathbf{FDQ}} \end{bmatrix} + \begin{bmatrix} S \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{n_{0dq}} \\ 0 \end{bmatrix} \tag{13}$$

where $\mathbf{R} = \text{diag}(\begin{bmatrix} r & r \end{bmatrix})$, $\mathbf{R_R} = \text{diag}(\begin{bmatrix} r_F & r_D & r_Q \end{bmatrix})$, and $\mathbf{S} = \begin{bmatrix} -\omega\lambda_q & \omega\lambda_d \end{bmatrix}^{\mathbf{T}}$.

We can write the voltage equation in simplified matrix form as

$$\mathbf{v_{dFDqQ}} = -\mathbf{R_{RS\omega} i_{dFDqQ}} - \mathbf{L} \dot{\mathbf{i}}_{\mathbf{dFDqQ}}, \tag{14}$$

where $\mathbf{v_{dFDqQ}} = \begin{bmatrix} v_d & -v_F & v_D = 0 & v_q & v_Q = 0 \end{bmatrix}^{\mathbf{T}}$, $\mathbf{i_{dFDqQ}} = \begin{bmatrix} i_d & i_F & i_D & i_q & i_Q \end{bmatrix}^{\mathbf{T}}$ while $\mathbf{R_{RS\omega}}$ and $\mathbf{L}$ are the following expressions

$$\mathbf{R_{RS\omega}} = \begin{bmatrix} r & 0 & 0 & \omega L_q & \omega k M_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega k M_F & -\omega k M_D & r & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix}$$

The state-space model for the currents is obtained by expressing $\dot{\mathbf{i}}_{\mathbf{dFDqQ}}$ from (14), i.e.

$$\dot{\mathbf{i}}_{\mathbf{dFDqQ}} = -\mathbf{L^{-1}} \cdot \mathbf{R_{RS\omega}} \cdot \mathbf{i_{dFDqQ}} - \mathbf{L^{-1}} \cdot \mathbf{v_{dFDqQ}} \tag{15}$$

D. Mechanical equation

The next step is to derive the mechanical part of the model [2]. The energy balance is written in the form

$$dW_{out} = dW_{Mech} - dW_{Field} + dW_{\Omega}, \tag{16}$$

where W_{Ω} is the energy losses in the resistance of the machine, W_{Field} is the energy of the field, W_{Mech} is the mechanical energy and W_{out} is the output energy of the synchronous generator. The time derivative of (16) is the power equation:

$$\frac{dW_{out}}{dt} = \frac{dW_{Mech}}{dt} - \frac{dW_{Field}}{dt} - \frac{dW_{\Omega}}{dt} \tag{17}$$

$$p_{out} = p_{Mech} - p_{Field} - p_{\Omega} \tag{18}$$

On the other hand, using (2), the output power of tri-phase system is:

$$p_{out} = \mathbf{v_{abc}^T i_{abc}} = \mathbf{v_{0dq}^T i_{0dq}} \tag{19}$$

The mechanical torque (T_{Mech}) is obtained by dividing power by the angular velocity $\omega = \frac{d\theta}{dt}$, i.e. $T_{Mech} = \frac{P_{Mech}}{\omega}$. This gives

$$T_{Mech} = \lambda_d i_q - \lambda_q i_d \tag{20}$$

From Newton's second law the equation of motion is

$$\frac{2H}{\omega_B}\dot{\omega} = T_{Mech} - T_{Electr} - T_{Dump}, \tag{21}$$

where T_{Mech} is the mechanical torque, T_{Electr} is the electrical torque per phase, H is the inertia constant and T_{Dump} is the dumping torque. The time and the rotation speed using per units i.e. dimensionless variables is $t_u = \omega_B t$ and $\omega_u = \omega/\omega_B$. Afterwards the normalized swing equation can be written as

$$2H\omega_B \frac{d\omega_u}{dt_u} = T_{Mech} - T_{Electr} - T_{Dump}, \tag{22}$$

where $2H\omega_B = \tau_j$, where τ_j is a time-like quantity coming from per unit notation not detailed here.

The electrical torque can be expressed from the flux and the current of the machine

$$T_{Electr} = \frac{1}{3}(\lambda_d i_q - \lambda_q i_d) \tag{23}$$

It is often convenient to write the damping torque as $T_{Dump} = D\omega$, where D is a damping constant.

The electrical torque expressed using the state vector of model (15) is then

$$3 \cdot T_{Elect} = \begin{bmatrix} L_d i_q \\ kM_F i_q \\ kM_D i_q \\ -L_q i_d \\ -kM_Q i_d \end{bmatrix}^T \cdot \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix} \tag{24}$$

Since the variables have a few orders of magnitude difference in their values in natural units, the equations are normalized with respect to a base value (corresponding to the normal range of the variables). This way all signals are measured in normalized units (p.u.). Using (22) and the definition of τ_j, the speed of the synchronous machine is

$$\dot{\omega} = \begin{bmatrix} -\frac{L_d i_q}{3\tau_j} \\ -\frac{kM_F i_q}{3\tau_j} \\ -\frac{kM_D i_q}{3\tau_j} \\ \frac{L_q i_d}{3\tau_j} \\ \frac{kM_Q i_d}{3\tau_j} \\ -\frac{D}{\tau_j} \end{bmatrix}^T \cdot \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \\ \omega \end{bmatrix} + \frac{T_{Mech}}{\tau_j} \tag{25}$$

Note, that (25) can be used as an additional state equation for state space model (15).

The loading angle (δ) of the synchronous generator is

$$\delta = \delta_0 + \int_{t_0}^{t} (\omega - \omega_r)dt$$

that we can differentiate to obtain the time derivative of the δ in per unit notation

$$\dot{\delta} = \omega - 1, \tag{26}$$

so the loading angle (δ) can also be regarded as a state variable in the state space model (15, 25). Altogether, there are 6 state variables: i_d , i_F , i_D , i_q , i_Q , ω and δ. The input variables (i.e. manipulatable inputs and disturbances) are: T_{Mech}, v_F, v_d and v_q. Observe, that the state equations (15, 25, 26) *are bilinear in the state variables* because matrix $\mathbf{R_{RS\omega}}$ in (15) depends linearly on ω.

E. Output equations

The output active power equation can be written in the following form:

$$p_{out} = v_d i_d + v_q i_q + v_0 i_0 \tag{27}$$

Assuming steady-state for the stationary components ($v_0 = i_0 = 0$), (27) simplifies to

$$p_{out} = v_d i_d + v_q i_q, \tag{28}$$

and the reactive power is

$$q_{out} = v_d i_q - v_q i_d. \tag{29}$$

Equations (28-29) are the *output equations* of the generator's state-space model. Observe, that these equations *are bi-linear in the state and input variables.*

F. Connecting the synchronous generator to an infinite huge network

Since every synchronous machine is connected to an infinite bus (shown in Fig. 4.) the next task is to extend the previous models with an infinite bus. In Fig. 4, resistance R_e and inductance L_e represent the output transformer of the synchronous generator and the transmission-line.

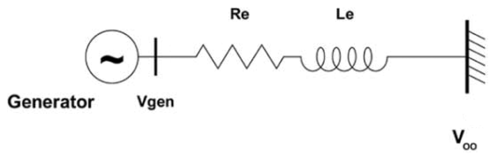

Fig. 4. Synchronous machine connected to an infinite bus

The matrix form of the modified voltage equation is as follows:

$$\mathbf{v_{abc}} = \mathbf{v_{\infty abc}} + R_e\mathbf{I_3 i_{abc}} + L_e\mathbf{I_3 \dot{i}_{abc}} \tag{30}$$

Equation (30) can be transformed to the $0dq$ coordinate system as

$$\mathbf{v_{0dq}} = \mathbf{Pv_{abc}} = \mathbf{Pv_{\infty abc}} + R_e\mathbf{I_3 i_{0dq}} + L_e\mathbf{I_3 \dot{i}_{0dq}} \tag{31}$$

The tri-phase voltage of the bus in the $0dq$ coordinate system is then

$$\mathbf{v_{\infty 0dq}} = \mathbf{Pv_{\infty abc}} = \sqrt{3}\mathbf{V_\infty} \begin{bmatrix} 0 \\ -\sin(\delta - \alpha) \\ \cos(\delta - \alpha) \end{bmatrix} \tag{32}$$

Afterwards, one can express the current vector $\mathbf{i_{0dq}}$ and voltage vector $\mathbf{v_{0dq}}$ as

$$\mathbf{P\dot{i}_{abc}} = \mathbf{\dot{i}_{0dq}} - \mathbf{\dot{P}i_{abc}} = \mathbf{\dot{i}_{0dq}} - \mathbf{\dot{P}P^{-1}i_{0dq}} \tag{33}$$

and

$$\mathbf{v_{0dq}} = \mathbf{v_\infty}\sqrt{3} \begin{bmatrix} 0 \\ -\sin(\delta - \alpha) \\ \cos(\delta - \alpha) \end{bmatrix} + \mathbf{R_e i_{0dq}} + \mathbf{L_e \dot{i}_{0dq}} - \omega\mathbf{L_e} \begin{bmatrix} 0 \\ -i_q \\ i_d \end{bmatrix} \tag{34}$$

The integration of resistance R_e and inductance L_e into voltage equation (14) can be done by a simple change in matrices $\mathbf{R_{RS\omega}}$ and $\mathbf{L}$.

The obtained voltage equation is:

$$\mathbf{v_{dFDqQ}} = \mathbf{\tilde{R}_{RS\omega} i_{dFDqQ}} + \mathbf{\tilde{L}\dot{i}_{dFDqQ}}, \tag{35}$$

where $\mathbf{v_{dFDqQ}}$, $\mathbf{i_{dFDqQ}}$, $\mathbf{\dot{i}_{dFDqQ}}$ $\mathbf{R_{RS\omega}}$ and $\mathbf{\tilde{L}}$ are

$$\begin{aligned} \mathbf{v_{dFDqQ}} &= \begin{bmatrix} v_d & -v_F & v_D = 0 & v_q & v_Q = 0 \end{bmatrix}^T \\ \mathbf{i_{dFDqQ}} &= \begin{bmatrix} i_d & i_F & i_D & i_q & i_Q \end{bmatrix}^T \\ \mathbf{\tilde{R}_{RS\omega}} &= \mathbf{R_{RS\omega}} + \text{diag}(\begin{bmatrix} R_e & 0 & 0 & R_e & 0 \end{bmatrix}) \\ \mathbf{\tilde{L}} &= \mathbf{L} + \text{diag}(\begin{bmatrix} L_e & 0 & 0 & L_e & 0 \end{bmatrix}) \end{aligned}$$

From (35) it is possible to express $\dot{\mathbf{i}}_{\mathbf{dFDqQ}}$ as

$$\dot{\mathbf{i}}_{\mathbf{dFDqQ}} = -\tilde{\mathbf{L}}^{-1}\tilde{\mathbf{R}}_{\mathbf{RS}\omega}\mathbf{i}_{\mathbf{dFDqQ}} - \tilde{\mathbf{L}}^{-1}\mathbf{v}_{\mathbf{dFDqQ}} \tag{36}$$

Now the extended state space model consists of Equations (36, 25, 26).

III. Model analysis

The above model has been verified by simulation against engineering intuition using parameter values of a similar generator taken from the literature. After the basic dynamical analysis, the set of model parameters is partitioned based on the model's sensitivity on them.

A. Generator parameters

The parameters are described only for phase a since the machine is assumed to have symmetrical tri-phase stator windings system. The stator mutual inductances for phase a are

$$L_{ab} = L_{ba} = -M_s - L_m \cos(2(\Theta - \frac{\pi}{6})) \tag{37}$$

where M_s is a given constant. The rotor mutual inductances are

$$\begin{aligned} L_{FD} &= L_{DF} = M_R, \\ L_{FQ} &= L_{QF} = 0 \\ L_{DQ} &= L_{QD} = 0 \end{aligned}$$

The phase a stator to rotor mutual inductances are given by: (from phase windings to the field windings)

$$L_{aF} = L_{Fa} = M_F \cos(\Theta) \tag{38}$$

where the parameter M_F is given.

The stator to rotor mutual inductance for phase a (from phase windings to the direct axis of the damper windings) is

$$L_{aD} = L_{Da} = M_D \cos(\Theta) \tag{39}$$

with a given parameter M_D.

The phase a stator to rotor mutual inductances are given by: (From phase windings to the damper quadratic direct axis)

$$L_{aQ} = L_{Qa} = M_Q \cos(\Theta) \tag{40}$$

Parameters L_d, L_q, M_D, M_F, M_R and M_D used by the state space model (15, 25, 26, 28,29) are defined as

$$\begin{aligned} &L_d = L_s + M_s + \tfrac{3}{2}L_m \\ &L_q = L_s + M_s - \tfrac{3}{2}L_m \\ &M_D = \tfrac{L_{AD}}{k} \\ &M_F = \tfrac{L_{AD}}{k} \\ &M_R = L_{AD} \\ &M_Q = \tfrac{L_{AQ}}{k} \\ &k = \sqrt{\tfrac{2}{3}} \end{aligned} \tag{41}$$

Figure 3 shows the position of the following (inductance and resistance) parameters in the simplified electrical circuit model:
L_F, L_D, L_Q, L_d, L_q, M_F, M_D, M_Q, r_F, r_D, r_Q,

Using the initial assumption of symmetrical tri-phase stator windings (i.e. $r_a = r_b = r_c = r$) we get the resistance of stator windings of the generator. The r_F represent the resistance of the rotor windings. The r_D and r_Q represent the resistance of the d and q axis circuit.

The resistance R_e and inductance L_e represent the output transformer of the synchronous generator and the transmission-line.

The parameters of the synchronous generator were obtained from the literature [1]. The stator base quantities, the rated power, output voltage, output current and the angular frequency are:

$$\begin{aligned} S_B &= 160 \text{ MVA}/3 = 53.333 \text{ MVA} \\ V_B &= 15 \text{ kV}/\sqrt{3} = 8.66 \text{ kV} \\ I_B &= 6158 \text{ A} \\ \omega_e &= 2\pi f \text{ rad/s} \end{aligned}$$

The parameters of the synchronous machine and the external network in per units are:

$L_d = 1.700$	$l_d = 0.150$	$L_{MD} = 0.02838$
$L_q = 1.640$	$l_q = 0.150$	$L_{MQ} = 0.2836$
$L_D = 1.605$	$l_F = 0.101$	$r = 0.001096$
$L_Q = 1.526$	$l_D = -0.055$	$r_F = 0.00074$
$L_{AD} = 1.550$	$l_Q = 0.036$	$r_D = 0.0131$
$L_{AQ} = 1.490$	$r_Q = 0.054$	$R_e = 0.2$
$V_\infty = 0.828$	$L_e = 1.640$	$D = 2.004$

B. Local stability analysis

The steady-state values of the state variables can be obtained from the steady-state version of state equations (36, 25, 26) using the above parameters. Equation (14) implies that the expected value to i_D and i_Q are 0, that coincide with the engineering intuition. The equilibrium point of the system is:

$$\begin{aligned} &\omega = 0.9990691 \\ &i_d = -1.9132609 \\ &i_q = 0.66750001 \\ &i_F = 2.97899982 \\ &i_D = -8.6242856 \cdot 10^{-9} \\ &i_Q = -5.3334899 \cdot 10^{-10} \end{aligned}$$

The state matrix $\mathbf{A}$ of the locally linearized state-space model $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ has the following numerical value in this equilibrium:

$$\begin{bmatrix} -0.0361 & 0.0004 & 0.0142 & -3.4851 & -2.5455 & -2.3285 \\ 0.0124 & -0.0049 & 0.0772 & 1.2011 & 0.8773 & 0.8025 \\ 0.0228 & 0.0044 & -0.0964 & 2.2057 & 1.6110 & 1.4737 \\ 3.5855 & 2.6464 & 2.6464 & -0.0361 & 0.0901 & 1.0247 \\ -3.5009 & -2.5839 & -2.5839 & 0.0352 & -0.1234 & -1.0005 \\ -8 \cdot 10^{-6} & -0.0002 & -0.0002 & -0.0008 & -0.0005 & -0.0011 \end{bmatrix}$$

The eigenvalues of the state matrix are:

$$\begin{aligned} &\lambda_{1,2} = -3.619088 \cdot 10^{-2} \pm \; j0.997704 \\ &\lambda_3 = -0.100024 \qquad \lambda_4 = -1.67235 \cdot 10^{-3} \\ &\lambda_5 = -4.724291e \cdot 10^{-4} \qquad \lambda_6 = -0.123426 \end{aligned}$$

It is apparent that the real part of the eigenvalues are negative but their magnitudes are small, thus the system is on the boundary of the stability domain.

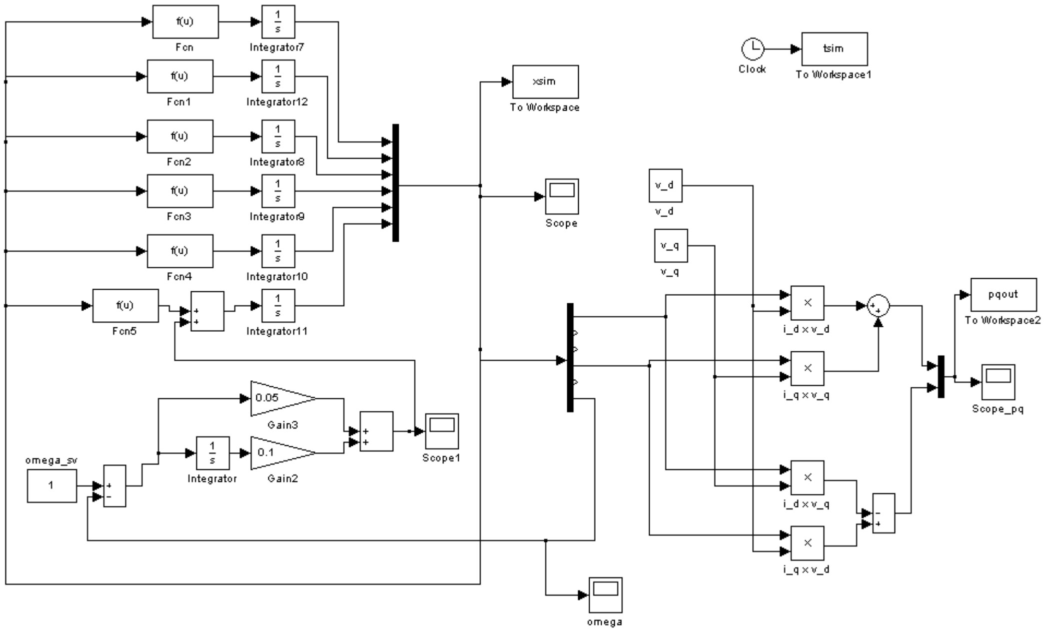

Fig. 5. The controller for the nonlinear model implemented in Matlab/Simulink

C. PI controller

The control scheme of the synchronous machine is a classical PI controller (Fig 5) that ensures stability of the equilibrium point under small perturbations [4]. The controlled output is the speed (ω), the manipulated input is the mechanical torque T_{Mech}. The proportional parameter of the PI controller of the speed is 0.05 and the integrator time is 0.1 in per units.

D. Model validation

The dynamic properties of the generator have been investigated in such a way that a single synchronous machine was connected to an infinite bus that models the electrical network (see Fig. 4). The response of the speed controlled generator has been tested under step-like changes of the exciter voltage. The simulation results are shown in Fig. 6, where the exciter voltage v_F and the torque angle δ are shown.

When the exciter voltage is increased the loading angle must be decreased as it can be seen in the Fig. 6.

E. Sensitivity analysis

The aim of this section define parameter groups according to the system's sensitivity on them.

Linkage inductances l_d, l_q, l_D, l_Q, L_{MD} and L_{MQ} are not used by the current model, only by the flux model [5]. It is not expected that the output and the state variables of system change when these parameters are perturbed, see Fig. 7. As it was expected, the model is insensitive for these parameters. Note, that the linkage inductance parameters are only used for calculating the fluxes of the generator.

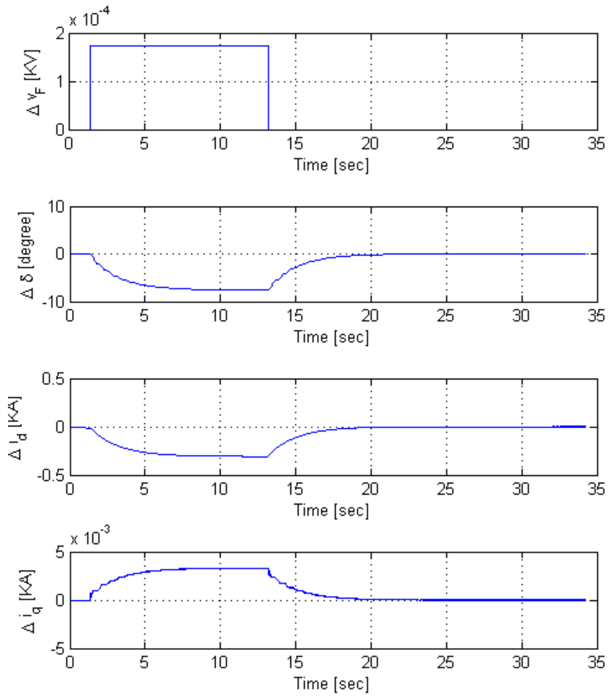

Fig. 6. Response to the exciter voltage step change of the controlled generator (Δ means the deviation form the steady-state value)

Sensitivity of the model to the controller parameters P and I and the dumping constant D has also been investigated. Since the PI controller controls for ω modifying the value of D the controller keeps ω at synchronous speed. This is why the output and the steady state value of the system variables do not change (as it is apparent in Fig 8) even

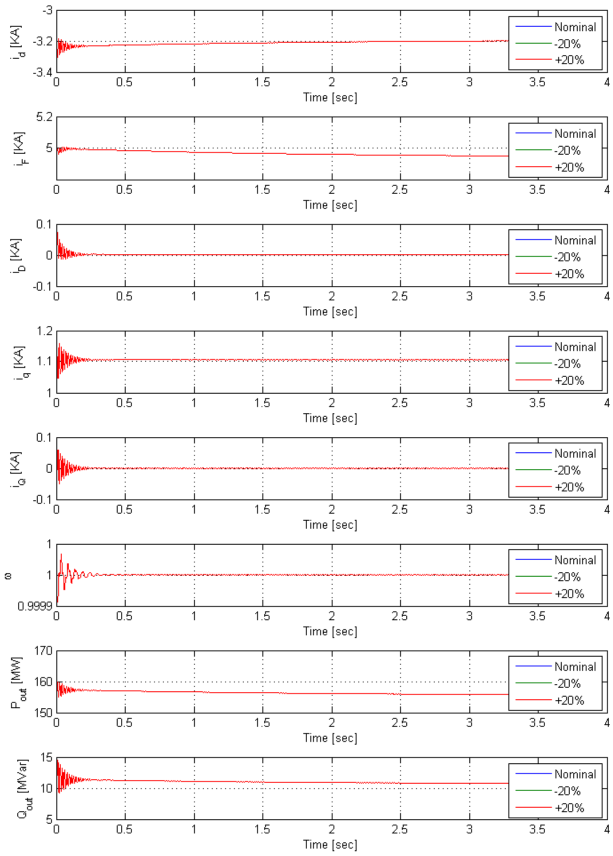

Fig. 7. The model states and outputs for a $\pm 20\%$ change of l_d

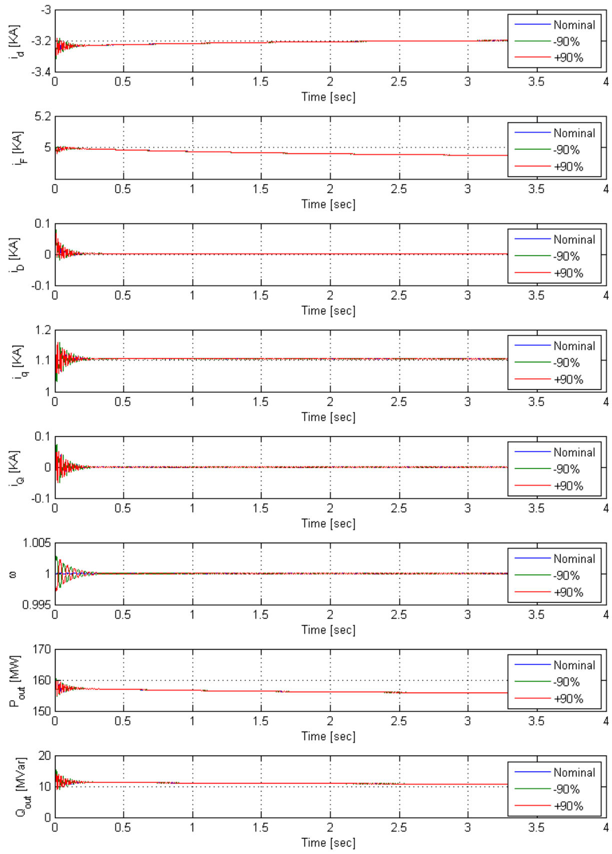

Fig. 8. The model states and outputs for a $\pm 90\%$ change of D

for a considerably large change of D.

Sensitivity analysis of the resistance of the stator and the resistance of the transmission line led to the same result. A $\pm 20\%$ perturbation in them resulted a small change in currents i_d, i_q and i_F, this causes the change of the effective and the reactive power of the generator, as it is shown in Fig. 9.

The analysis of the rotor resistance r_F showed, that the $\pm 20\%$ perturbation of r_F kept the quadratic component of the stator current (i_q) constant, but currents i_d and i_F were changed. The output of the generator also changed, as it is shown in Fig. 10.

The sensitivity of the model states and outputs to the inductance of the rotor (L_F) and the inductance of the direct axis (L_D) has also been analyzed. The results shows only a moderate reaction in i_d and i_F to the parameter perturbations, and the equilibrium state of the system kept unchanged. However, decreasing the value of the parameters to the 90 percent of their nominal value destabilized the system. The results of a $\pm 9\%$ perturbation in L_F are shown in Fig. 11. A small perturbation of the outputs is noticeable.

Finally, the sensitivity of the model (15, 25, 26, 28,29) to the linkage inductance L_{AD} has been examined. When the parameter has been changed $\pm 5\%$, currents i_d and i_F changed only a little. On the other hand, the steady-state of the system has shifted as it can be seen in Fig. 12. A parameter variation of 5% destabilized the system.

As a result of the sensitivity analysis, it is possible to define the following groups of parameters:

- Not sensitive
  inductances l_d, l_q, l_D, l_Q, L_{MD}, L_{MQ}, L_{AQ}, L_Q, damping constant D and the controller parameters P and I. Since the state space model of interest is insensitive for them, the values of these parameters cannot be determined from measurement data using any parameter estimation method.
- Sensitive
  - Less: resistances of the stator r and the transmission-line R_e.
  - More: resistance r_F of the rotor and the inductance of transmission-line L_e. These parameters are candidates for parameter estimation.
- Critically sensitive
  linkage inductance L_{AD}, inductances L_D and L_F. These parameters can be estimated very well.

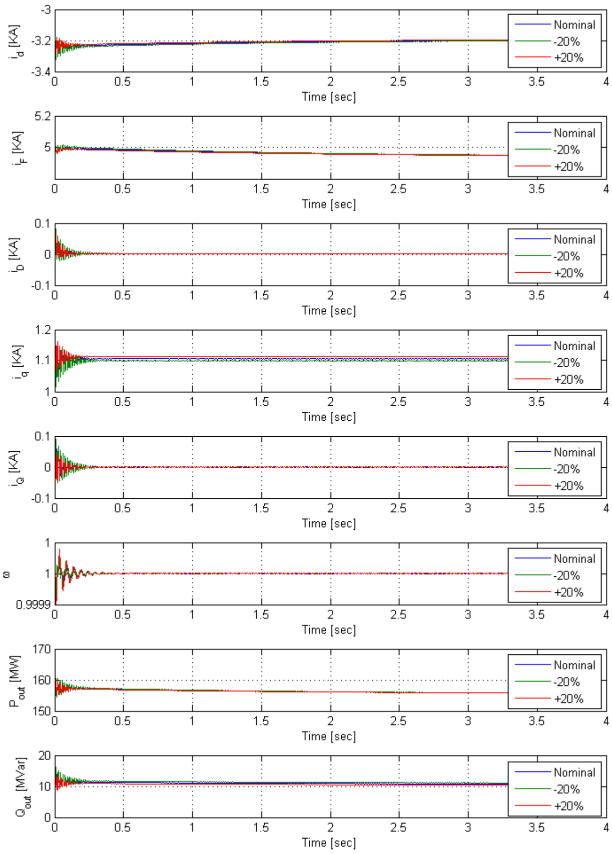

Fig. 9. The model states and outputs for a $\pm 20\%$ change of r_{resist}

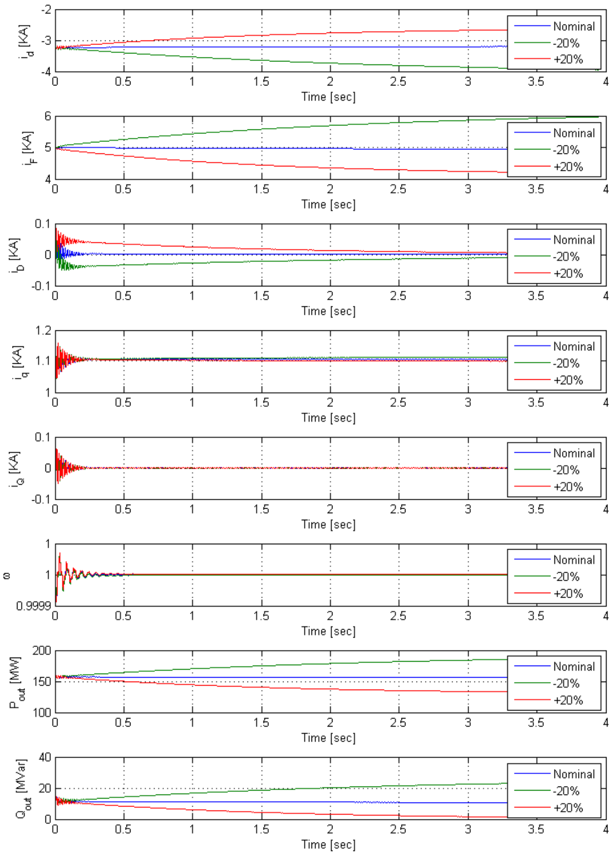

Fig. 10. The model states and outputs for a $\pm 20\%$ change of r_F

IV. Conclusion and further work

The simple bilinear dynamic model of an industrial size synchronous generator operating in a nuclear power plant described in [5] has been investigated in this paper.

It has been shown that the model is locally asymptotically stable around a physically meaningful equilibrium state with parameters obtained from the literature for a similar generator. The effect of load disturbances on the partially controlled generator has been analyzed by simulation by using a traditional PI controller. It has been found that the controlled system is stable and can follow the setpoint changes in the effective power well.

Eighteen parameters of the system has been selected for sensitivity analysis, and the sensitivity of the state variables and outputs has been investigated for all of them. As a result, the parameters has been partitioned to four groups.

Based on the results presented here, the further aim of the authors is to estimate the parameters of the model for a real system from measurements. The sensitivity analysis enables us to select the candidates for estimation that are r_F, L_e, L_{AD}, L_D and L_F.

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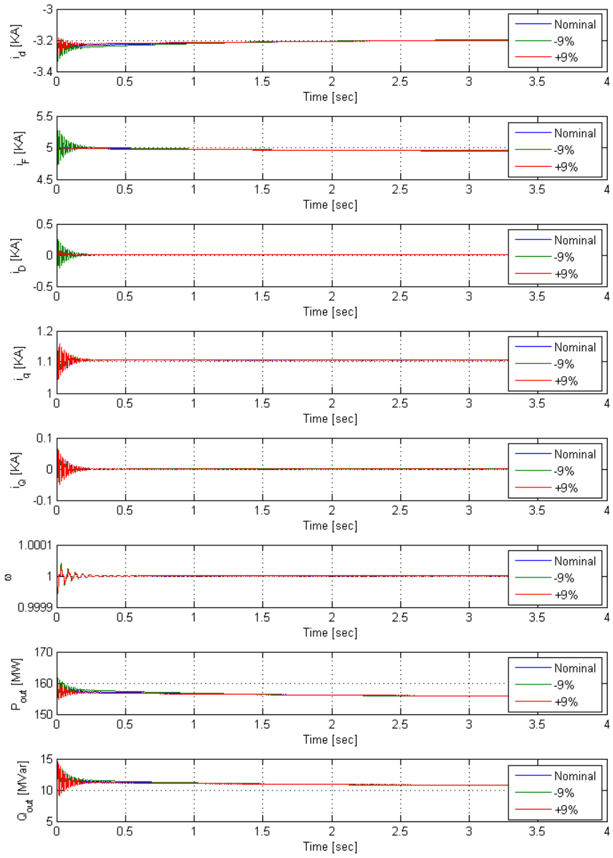

Fig. 11. The model states and outputs for a $\pm 9\%$ change of L_F

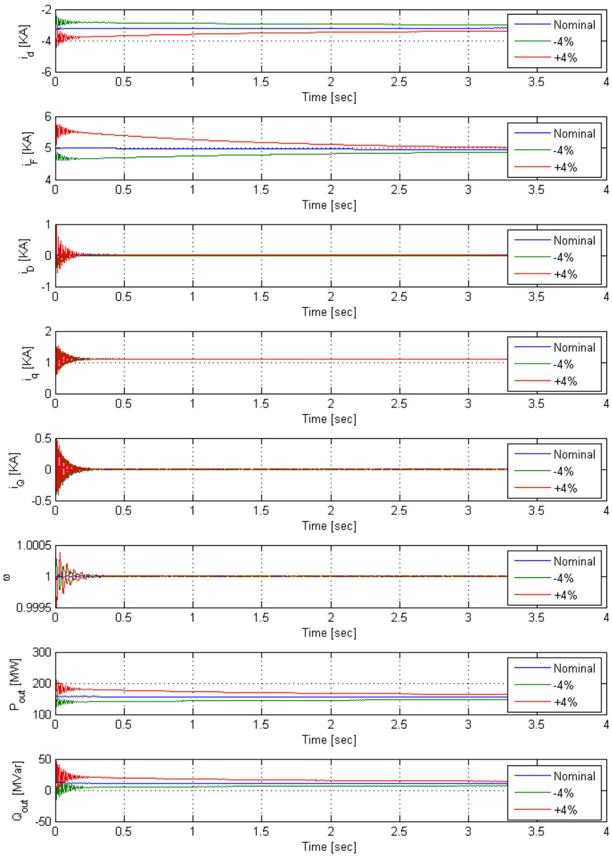

Fig. 12. The model states and outputs for a $\pm 4\%$ change of L_{AD}