

A one-sided flow based combinatorial auction electricity market model

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Abstract: A new market model based on combinatorial auction for electricity trade is introduced, where the central authority (CA) conducting the auction is integrated with the transmission system operator (TSO). The TSO receives the consumer demands and prices, and conducts a flow based combinatorial auction for the generators. During the auction the transmission limits of the underlying power network are taken explicitly into account, thus the transmission capabilities are utilized efficiently. The model enhances most efficient utilization of the concave production characteristics of the generators. We show that in this concept the TSO is motivated for network expansion.

Key-Words: Electricity Markets, Combinatorial Auctions

1 Introduction

1.1 Market and auction models in electrical energy trade

Due to the challenging and important nature of the problem, market concepts of electricity have been widely studied [21, 25], and the application of Operations research already provided impressive results in the field [1]. A significant part of the electricity-market literature focussing on day ahead markets, uses auction concepts.

As described by [7], there are several characteristic aspects of electricity auctions, which separate them from the vast body of general auction theory. The most obvious of these aspects is the structure of generation costs (e.g. start-up costs, no-load costs, etc.). Demand and cost uncertainties, which are also typical features of this sector, are discussed in the paper by [22]. [7] discuss the efficiency of multi-unit auctions, focussing on the sequential decomposition of the auction process, and show that the unique Subgame Perfect Nash Equilibrium outcome of the horizontal auction is efficient. A centralized power pool auction, based on multipliers stabilization procedure is described in [14]. [24] analyzes the bidding behavior of capacity-constrained firms in an electricity spot market. An application using real market data is provided by [6].

In general, even if no transmission constraints are

taken into account, the problem may be highly complex in several different levels [7, 16, 8, 9]. Other models focus on scenarios, where the limited transmission capacity of the underlying physical power network comes into play.

In available transmission capacity (ATC) models [12] on the one hand there are internal markets, where the volume of trading is small enough (or the transmission capacity of the internal network is high enough) to say that the network transmission constraints never apply, and on the other hand bottlenecks are present between these internal markets, which usually correspond to state borders. The effect of integration of energy markets on market power is discussed in [20]. If there are differences in the market clearing prices between the internal markets, the bottleneck transmission capacity rights are auctioned by the transmission system operator (TSO) to ensure that these lines are not overloaded. Trivially, as the bottleneck gets tighter, the higher price the auction will result in. This leads to the thwarting situation that from a certain point of view TSOs may be interested in low transmission capacities of the power network.

Flow-based methods in general use the Power Transfer Distribution Factor (PTDF) matrix to describe how much load a transaction puts on a certain network component. The PTDF matrix can be calculated by network parameters, and it is provided by the TSO. The auction income distribution of flow-based explicit auctions is discussed by [17]. [10] per-

form the sensitivity analysis of a flow-based capacity allocation, proposed by the central allocation office (CAO) in the Central-Eastern European region. [19] provide a model which in addition to network constraints, also accounts for transmission losses.

The unit commitment approach aims to find the least-cost commitment and dispatch of a set of generating units to meet expected load over a time horizon consisting of a fixed number of periods, typically, twenty-four single-hour periods [25]. A case study of centralized unit commitment is described by [4, 3].

1.2 Motivation

Despite the liberalization-oriented reform-environment in the European Union [13], trends towards centralized regulation of energy prices are occasionally still prevalent in the Central-European Region [27]. In our model we do not assume a market clearing price, rather we account for individual zonal prices. This approach may be plausible e.g. if the central authority wishes to guarantee low prices of electrical energy for domestic consumers, or grants it at discount prices to accentuated industrial consumers. A market, in which consumers preliminary and explicitly form their bids ('I need X MWs at a price of Y') result in the same scenario.

In this paper we present a one-sided combinatorial auction mechanism [5] which accounts for network transmission constraints, and suppose that the authority conducting the auction is integrated with the TSO. We do not use the PTDF matrix, rather we show that directly incorporating the network calculations in the auction model is possible.

The one-sided concept is motivated by the fact, that because of their concave production cost characteristics, generators are the key players of the market, thus the concept of the combinatorial auction for generators is straightforward for enhancing the efficient utilization of their production capabilities.

Furthermore we wish to show that in contrast to ATC market models, where the TSOs may be motivated in bottlenecks, since these are auctioned, in the proposed model the central authority (integrating the TSO) is interested in high transmission capacity of the network.

2 Materials and Methods

In the current paper, we focus on the utilization of generation capacities, rather than on the implications of start up costs. For the aim of simplicity we assume that the consumption demands are to be satisfied only at one time period. Later we discuss, how the model

may be extended to multiple consecutive time periods (in other words horizontally).

2.1 Models of the physical components

2.1.1 The DC load flow model of the power transmission network

DC load flow models are representative in the power system economics literature (see e.g. [26, 28, 23]). The linear nature of these models allows their application even in the case of larger systems. In this paper, it is assumed that every node of the energy transmission network is assigned either to a generator or a consumer.

The power transmission system is described by a graph, the *system graph*, in which n nodes (or buses) are connected by m edges, which naturally represent the transmission lines. We assume n_g generators, and $n_c = n - n_g$ consumers.

The details of the DC load flow model are described in [2]. One of the most important properties of the DC load flow model is that given a power injection vector P , the network topology and line parameters, the flows (q_{ij}) can be uniquely determined via linear equations. We assume that the first n_g elements of P correspond to generators for which the p_i values are negative. P_g will denote the truncated vector, which holds only the first n_g elements of P . During the optimization process, these elements of P will represent the decision variables, while the remaining elements corresponding to the consumption values will be constant. P_c will denote the remaining part of P corresponding to consumers.

2.1.2 Model of the generators

As described eg. in [7], a generator's costs to produce energy fall into two general categories; there is a fixed "start-up" cost incurred each time a generating plant is turned on, and variable cost per GWh once the plant is up and running. There exists an inverse relationship between the start-up cost associated with a technology and its variable cost (for example, a nuclear plant has a large start-up cost but relatively small variable cost per GWh, while a gas-steam turbine has a relatively low start-up cost, but incurs a large variable cost per GWh). Generation plants have a constraint on the maximum number of GW they generate at any point in time and are unable to store electricity, but have few restrictions on the duration for which they can generate. Furthermore, as nicely worded by [7], generators in addition to their various costs, are subject to intertemporal dispatch constraints that relate their output in different time periods. These characteristics

create cost dependencies in intertemporal production so that the average cost of generating a fixed amount of electricity varies with the dispatch schedule and number of units generated. Regarding the generated quantity, generation costs for power plants are typically non-convex in the normal operational range. In other words, typically, more the total production capacity of a plant is utilized, less the production price of one unit of energy becomes.

In the proposed model, we focus on the concave generation costs. As for the sake of simplicity, the proposed model will have only one time period, initially we neglect the start-up costs, however we will discuss how these factors may be incorporated in a possible future extension of the model. Regarding the generation costs, we assume linearly decreasing marginal production characteristics for each generator: 1

$$c_j(p_j) = a_j - m_j p_j \quad (1)$$

where a_j [\$/MWh] and $m_j > 0$ [\$/MWh²] are the constants describing the production characteristics of generator j (which depend on the applied technology), while p_j [MWh] is the total power produced by the generator j . The total generation cost of a generator can be formulated as: $C_j = c_j(p_j)p_j$. The vector C holds the generation costs.

2.2 Model of the auction process

Block orders in electricity auctions allow generators to offer cheaper prices for delivery in multiple consecutive hours, and this way spread out their start-up cost. Because the presence of block orders, multi unit electric energy auctions are often regarded as combinatorial auctions [18]. It is true that block orders (which represent the 20% of total trading on some exchanges according to [18]) do have the fill-or-kill (or all-or-nothing) property, which means that binary variables are necessary to model the auction problem, and in this sense they are analogous to combinatorial auction problems. On the other hand, while in the classical combinatorial auction problem [5] each bidder may place a bid to every subset of the goods, on the most power exchange mutually exclusive bids are not allowed. In other words, if a generator has 200 MW generating capacity, it can not submit a 100 MW and a 150 MW block order for the same period. In our auction, generators place bids on every subset of the demands.

We call our auction one sided, since we assume that the consumption demands and the prices the consumers pay for their amount is given and announced, and the generators bid at the central authority (CA), integrated with the TSO, to fulfill these needs. The

integration of the TSO is necessary to take network transmission constraints into account. Similar to [22, 16, 25, 9], price-inelastic demand is assumed.

Our model for the auction uses the classical formalism based on [5], namely

$$\begin{aligned} \max \quad & \sum_{j \in N_g} \sum_{S \subseteq M} b^j(S) x(S, j) \\ \text{st.} \quad & \sum_{S \ni i} \sum_{j \in N_g} x(S, j) \leq 1 \quad \forall i \in M \\ & \sum_{S \subseteq M} x(S, j) \leq 1 \quad \forall j \in N_g \\ & x(S, j) = 0, 1 \quad \forall S \subseteq M, \quad j \in N_g \end{aligned} \quad (2)$$

where $b^j(S)$ is the bid that agent j has announced it is willing to pay for the S subset of the goods (M). N_g is the set of generators (or players/bidders). $x(S, j) = 1$ if the bundle $S \subset M$ is allocated to $j \in N_g$ and zero otherwise. In our case, M is the set of all consumption demands. If we take only nonempty subsets into account, $|S| = 2^{n_c} - 1$, where $n_c = N$ is the number of consumers (each with one demand for the aim of simplicity). Let us denote the length of vector x by l_x , which is equal to n_g times $|S|$. b is composed of $[b^1, b^2, \dots, b^N]$ where b^j is a vector of length $|S|$ for all j , containing the bids corresponding to all possible S for each player j .

The first set of equality conditions in 2 corresponds to the requirement that no overlapping subsets are assigned. The second constraint ensures that no bidder receives more than one subset.

For a certain subset of the consumption needs, the generators are able to calculate their efficiency and generating price in the forthcoming generation process, based on their production characteristics. For every subset, a generator can evaluate its total production in the case that subset is assigned to it, and thus is able to determine the production price per unit. Every generator takes this data into account while placing bids on the subsets (S) of consumption demands (M)¹. The exact value of the bid is determined by the profit it aims to reach.

Since the production capacity of every generator is limited, there may be subsets, for which the total needed generation value exceeds the production limit.

¹Two very simple examples: If there is a demand of p_1 MWh-s, at the price of a_1 and a generator is able to provide this p_1 MWh-s at a price $c_1 < a_1$ (depends on the applied technology), it will place a bid b to the corresponding one-element subset such that $0 < b < a_1 - c_1$. When bidding on a subset containing p_1 and p_2 on prices a_1 and a_2 respectively, and the generator is able to produce the amount $p_1 + p_2$ at the price c_{12} the bid b will be $0 < b < a_1 + a_2 - c_{12}$.

For these subsets, generators bid 0. The other possibility for zero bid is the case, when the announced price of a certain consumer for its demand is not high enough to cover the production cost in itself. Of course, while the bid on this one-element subset will be 0, bids on other subsets including this demand may be greater than 0, because as the total production amount rises, the production price per unit decreases (as implied by the concave production characteristics).

The profit of a generator may be calculated as

$$\vartheta(j) = I^j - \sum_{S \subseteq M} b^j(S) y(S, j) - GC^j(x) \quad (3)$$

where I^j is the income of generator j , b is the vector of bids, and $GC^j(x)$ is the production cost of generator j , in the case of the auction outcome x .

Let us denote the power injection vector of the network with P . A p_i in $\{1, \dots, n\}$ value is negative if power is injected at the node (generators) and positive if power is consumed there. The matrix W represents the connection between any possible outcome of the auction, and the power injection vector.

$$P = Wx \quad (4)$$

Each column of the matrix $W \in \mathbb{R}^{n \times l_x}$ corresponds to an element of the vector x . E.g. the first column of W determines the implied power values, if the first subset of consumption demands is assigned to generator one. Following [2], the network constraints may be written in the form of

$$|B^D A^T B^+ P| < \bar{Q} \quad (5)$$

where the vector \bar{Q} holds the maximal line power loads, and matrices A , B^+ , and B^D are determined by the network topology and parameters. The absolute value corresponds to the consideration that the flow of a given line can not exceed its limit in neither direction. The absolute values can be removed from the problem e.g. as described by [15].

In this way, network capacity constraints in the DC load flow model may be reformulated using the variables of the combinatorial auction problem.

3 Results

3.1 The example network

We demonstrate the operation of the proposed method and analyze its properties on the network example depicted in Fig. 1. Nodes 1, 2 and three represent generators, while nodes 4,5,6 and 7 are the consumers.

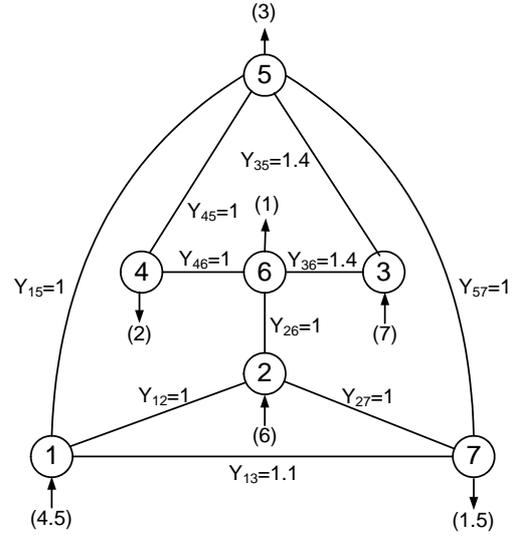


Figure 1: The example network. The Y values correspond to the admittance of lines, the in and out-bound arrows at nodes indicate generation and consumption respectively. The numerical values in parentheses at the nodes show the generation capacities and consumption demands.

We assume that the three generators' production characteristics are described by the curves depicted in Fig. 2

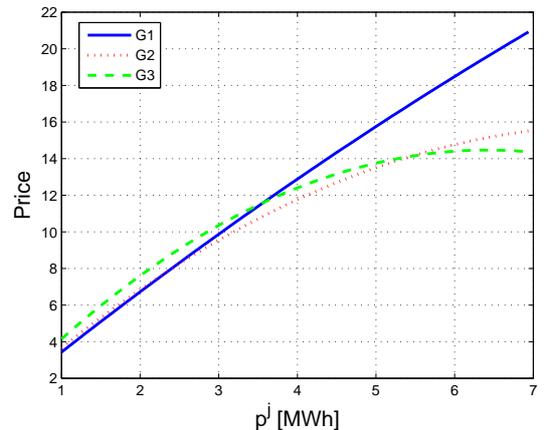


Figure 2: The production characteristics of generators. p^j is the amount of energy generated by generator j .

3.2 A high throughput network favors efficient production and results in higher income of the central authority

In our first simulation study we wish to analyze the effect of network congestion on the auction outcome, thus suppose that the generators use a constant, very simple bidding strategy. We assume that generators calculate their potential profit for each S subset of the 4 consumption demands, and if they are able to provide the corresponding quantity at a production cost which is less than the potential income determined by the prior given prices, they bid the 30% of their potential profit (in other words they sacrifice 30% of their profit for the central authority in the auction). The prices per unit corresponding to the consumption demands are [4 4.25 4.33 3.92] (regarding nodes 4,5,6 and 7 respectively).

Let us shortly discuss the results summarized in Table 1. The first row corresponds to the case when generator 1 serves consumer 5, and generator 3 serves consumers 6 and 7. As we can see, in this case the network transmission capacity is so low that not every consumption demand is satisfied (consumer 4 is not served). This is the reason for the low total generation cost in this case (compared to the following rows). It is possible to include constraints in (2) to ensure that all demands are satisfied (if the network allows this), but in this case some of the resulting generator profits may be negative. Of course, in such cases of regulations, these generators have to be compensated. At $\bar{q}/Y = 1.5$ and 1.75 we can see that generator 2 operates at the total load of 3.5 and 4.5 MWs, in which range its operational cost is lowest of the three (see Fig. 2). As the transmission capacity further increases, generator 3, which is capable the most efficient production at high loads, seizes three consumers for itself, and utilizes its capacities the most efficient way. generator one is efficient at low loads, thus often acquires single consumer demands. The increase in production efficiency implies a higher profit, and thus a higher bid, which is reflected in the total income (I_c) which increases steadily with the transmission capacity. This result underlines that in the proposed model, the TSO is not interested in the presence of bottlenecks. Furthermore let us note that from $\bar{q}/Y = 1.5$ (when already all consume demands are served), the total generation cost decreases, the production becomes increasingly efficient.

4 Discussion

In real life scenarios, start-up/shutdown, or ramp-up/down and no-load costs contribute significantly to

the total generation cost. Basically, the model is capable to be temporally extended. The most simple approach is to simply compose M as the set of consumption demands with time parameters (that is the start time and end time of the given consumption demand). This way, when generators bid to a given subset of consumption demands, they calculate their time-dependent production schedule, and the implied start-up and shut-down costs and determine their bids according to this data. The benefit that bids may be placed to mutually exclusive demands (e.g. when two demands individually are feasible, but satisfying both would exceed the production limit of the generator at certain time) still holds, however the decomposition of production capacity and transmission constraints for each time instance is needed. The current manuscript aims to highlight the basic idea of the proposed concept, and demonstrate, how the individual production characteristics come into play in the model.

The hourly-detailed approach on the other hand, may represent a greater challenge for the algorithm. If we do decompose e.g. a shoulder load demand [7] to 6-8 distinct hours, the cardinality of S , and thus the number of variables in the combinatorial auction increases exponentially. According to this observation, the temporal extension of the proposed approach may only be used efficiently, if continuous consumption demands are not decomposed to consecutive parts, which are to be marketed individually. Whether, and under what conditions this approach serves the consumers interests, should be the subject of a future study.

In addition, the proposed model is capable of testing and comparing different agent-based methods regarding bidding strategies - see [11].

5 Conclusions

We presented a possible combinatorial auction mechanism for electrical energy market, which takes network constraints into account. The generators in these mechanism may place bids for each subset of the consumption demands, enhancing the efficient utilization of production capabilities. Assuming equally profit seeking generators, we have shown that the increase of network transmission capacity results in more efficient production, and increasing income of the TSO.

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\bar{q}/Y	I_c	ϑ^1	ϑ^2	ϑ^3	$\sum \vartheta$	$\sum GC$	Λ
1.25	1.20825	2.016	0	0.80325	2.81925	18.9325	{5} {0} {6,7}
1.5	1.941	0.63	2.219	1.68	4.529	24.49	{6} {4,7} {5}
1.75	2.376	0	3.864	1.68	5.544	23.04	{0} {5,7} {4,6}
2	2.751	0	6.139	0.28	6.419	21.79	{0} {5,6,7} {4}
2.5	3.44025	0.55125	0	7.476	8.02725	19.4925	{7} {0} {4,5,6}
3	3.92025	0.63	0	8.51725	9.14725	17.8925	{6} {0} {4,5,7}

Table 1: The change of central income (I_c) the profit of single generators (ϑ^j), the total profit of generators ($\sum \vartheta$) and the total generation cost ($\sum GC$) as the power transmission constraints get loose. We suppose that the maximal transmission (\bar{q}) on each line is linearly dependent on its admittance value (Y). Λ is the lexicographic enumeration of Λ^j -s, where Λ^j denotes the consumer subsets assigned to each generator.

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