Modeling of emission in urban traffic networks

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Abstract

This work suggests a framework for modeling the traffic emissions in urban networks that are described by the network fundamental diagram (NFD) concept. The approach is the same as in [Csikós et al. (2012)]: traffic emission is formalized in finite spatiotemporal windows as a function of aggregated traffic variables (i.e. total travel distances in the network and network average speed). The framework is extended for the size of an urban network during a signal cycle - the size of a window in which the network aggregated parameters are modeled in the NFD concept. Simulations are carried out for model accuracy analysis, using the microscopic Versit+Micro model as reference. By applying the emission model function and the traffic modeling relationships, the control objective for pollution reduction is formalized as a regulator problem for a model predictive control framework.

1 Introduction

The concept of the protected network PN has been highlighted recently as an efficient solution to prevent traffic jams in certain urban subnetworks. A protected network usually represents a city center or a dense urban area that needs protection against insatiate demands during rush hours. The traffic performance of a PN is basically managed by perimeter control, allowing an optimal traffic flow through the network gates. The traffic control of a PN is often related to the theory of the urban fundamental diagram which was first proposed by [Godfrey (1969)]. The theory is called both macroscopic fundamental diagram (MFD) and network fundamental diagram (NFD). The NFD concept has been widely investigated during the past decades, e.g. [Mahmassani et al. (1987), Daganzo et al. (2008)].

The applicable control methodologies have also appeared by using the traffic lights along the perimeter of the PN as controllable gates. Daganzo introduced a control rule based on time dependent switching conditions [Daganzo (2007)]. The work [Jong et al. (2013)] analyzes the effect of different signal strategies within the PN on the shape of the NFD, and proposes a control system, separating the control along the links at the boundary of the PN, and inside the PN. [Keyvan-Ekbatani et al. (2013)] provides a thorough description of the NFD model, considering the actuation dynamics. A linear feedback regulator control (PID control) is designed for the described model dynamics.

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So far, the framework of protected networks has been only used to optimize traffic performance within the network as the sole design objective. However, modern transport engineering practice requires a design for optimizing emissions of road traffic. For this end, the emission of urban networks needs to be modeled with an important condition: the emission modeling framework has to use the measurements of the existing traffic system model (i.e. no further measurements can be required).

In this work, a framework is suggested and analyzed for the modeling of pollutant emissions in urban networks. The model is based on the macroscopic traffic emission framework, introduced in [Csikós et al. (2012)]. In that work, the emission of traffic flow is formalized as a bivariate function of time and space (i.e. a distributed parameter system), as a function of the macroscopic traffic variables. By using the emission modeling framework, control objective is stated for emission optimization. The traffic system model, used in our work highly relies on the system model proposed in [Keyvan-Ekbatani et al. (2013)] with certain modifications: with extension of system states by modeling the queuing dynamics, and simplifications on the actuation delays. The paper is organized as follows. First the mathematical model of the system is stated in Section 2. The modeling of urban emissions is suggested in Section 3. The accuracy of the model is analyzed in two case studies in Section 4. Then, the control objective for pollution reduction is stated in Section 5.

2 Traffic model

For the model, basically the same assumptions are taken as in [Keyvan-Ekbatani et al. (2013)] with certain modifications.

2.1 Model equations

The basic and most important rule that has to be satisfied by a traffic network is the conservation law. For the protected network *PN*, it can be formalized as follows:

$$N_{PN}(k+1)=N_{PN}(k)+T_s\left[Q_{in}(k)+Q_d(k)-Q_{out}(k)\right] \tag{1}$$

where k denotes the discrete time step index and T_s in unit [h] is the discrete sample time step, in our case the signal cycle time. Practically, Eq. (1) depicts the state variation during the time interval $[kT_s,(k+1)T_s]$. State variable is represented by $N_{PN}(k)$, the number of vehicles within the protected network, given in passenger car equivalent [PCE] (the different types of road vehicles can be expressed in the ratio of private car). $Q_{in}(k)=\sum_{j=1}^{n_{in}} q_{in,j}(k)$ in unit [PCE/h] is the sum of inflow of vehicles to the protected network, whereas n_{in} denotes the number of controlled gates. $Q_d(k)=\sum_{j=1}^{n_d} q_{d,j}(k)$ is the sum of uncontrolled inflow, and n_d denotes the number of uncontrolled gates. $Q_{out}(k)=\sum_{j=1}^{n_{out}} q_{out,j}(k)$ is the sum of outflow of vehicles from the protected network, with n_{out} denoting the number of exit gates.

The second basic law that is used for the model is the concept of urban NFD (see [Gartner et al. (2004), Daganzo et al. (2008)]). The NFD of the protected network *PN* describes the relationship between the total travel distance (TTD_{PN}, in unit $[PCE{\cdot}km]$) and the total time spent (TTS_{PN}, in unit $[PCE{\cdot}h]$) within the protected network (during a discrete step). These traffic variables can be obtained by following the concept of [Keyvan-Ekbatani et al. (2013)], reformalized using [Ashton (1966)]:

$$TTS_{PN}(k) = T_s\sum_{j=1}^{n_{link}} N_j(k) \tag{2}$$

$$TTD_{PN}(k) = T_s\sum_{j=1}^{n_{link}} q_j(k)L_j \tag{3}$$

where n_{link} denotes the number of links in the protected network, $q_j(k)$ and L_j in unit [km] denote the traffic flow and length of link j respectively.

A full information control is assumed, i.e. that all traffic variables are measured and the exact number of vehicles in the protected network can be calculated based on the following equation:

$$N_{PN}(k) = \sum_{j=1}^{n_{link}} \rho_j(k) L_j \tag{4}$$

where $\rho_j(k)$ in unit [PCE/km] denotes the traffic density on link j. Using (4) the knowledge of the operational NFD can be supposed.

The fundamental relationship can be stated as follows:

$$TTD_{PN}(k) = F\left(TTS_{PN}(k)\right) + \varepsilon(k) \tag{5}$$

where $F(\cdot)$ denotes the nonlinear function of the complete operational NFD, fitted to historic measurements. $\varepsilon(k)$ denotes the fitting error, considered as noise in the system dynamics.

The network model also assumes that the total outflow $Q_{out}(k)$ of the protected network is proportional to $TTD_{PN}(k)$, satisfying the following equation:

$$Q_{out}(k) = \Gamma \frac{TTD_{PN}(k)}{LT_s} \tag{6}$$

where $0 \leq \Gamma \leq 1$ is the network exit rate parameter, L is the average link length in the protected network. Coefficient Γ can be fitted using the measurements of the total outflow of the network and TTD_{PN}.

The actuator dynamics of traffic lights at controlled gate j is described by Eq. (7)

$$q_{in,j}(k) = \beta_j q_{g,j}(k) \tag{7}$$

where β_j is the portion of gated flow ($q_{g,j}$) that enters the PN, ($0 \leq \beta_j \leq 1$) and can be approximated by fitting to the measurements of $q_{in,j}$ and $q_{g,j}$.

In addition to the model outlined in [Keyvan-Ekbatani et al. (2013)], our framework considers the queuing dynamics as well:

$$l_j(k+1) = l_j(k) + T_s\left(q_{dem,j}(k) - q_{in,j}(k)\right) \tag{8}$$

where l_j in unit [PCE] denotes the queue length at gate j where the traffic demand $q_{dem,j}$ emerges.

To sum up the differences between the proposed model and the one used in [Keyvan-Ekbatani et al. (2013)], for modeling simplifications, in our work a full information control system is supposed, i.e. the complete operational fundamental diagram is supposed to be known. Thus, following the notations in [Keyvan-Ekbatani et al. (2013)], for the estimation error ε_1=0 and for the operational fundamental diagram correction factors A=1 and B=1 are supposed. Apart from this, by placing the gates at the boundary of the protected network, the delay of the actuator system is eliminated and τ=0 is supposed. The model, however, is extended by modeling the queues at the gates to optimize inflow allocation.

2.2 State-space model

In this section the model equations are reformulated in a nonlinear state-space system framework. The dynamic equation of the system can be stated as follows:

$$\begin{pmatrix} N_{PN}(k+1) \\ l_1(k+1) \\ \dots \\ l_{n_{in}}(k+1) \end{pmatrix} = \begin{pmatrix} N_{PN}(k) - T_s \frac{\Gamma}{L} F(N_{PN}(k)) \\ l_1(k) \\ \dots \\ l_{n_{in}}(k) \end{pmatrix} + \begin{pmatrix} T_s \sum_{j=1}^{n_{in}} q_{in,j}(k) \\ -T_s q_{in,1}(k) \\ \dots \\ -T_s q_{in,n_{in}}(k) \end{pmatrix} + \begin{pmatrix} T_s \sum_{j=1}^{n_d} \bar{q}_{d,j}(k) \\ T_s q_{dem,n_1}(k) \\ \dots \\ T_s q_{dem,n_{in}}(k) \end{pmatrix} \tag{9}$$

In the equation, the second and third terms include the effect of control inputs and disturbances, respectively.

2.3 System variables

As *state variables*, the vehicle number in the protected network N_{PN} and queue lengths of the controlled gates $l_1 ... l_{n_{in}}$ are considered:

$$x(k) = [N_{PN}(k), \;\; l_1(k), \ldots, l_{n_{in}}(k)]^T \in \mathbb{R}^{n_{in}+1} \tag{10}$$

The *disturbances* are collected in the following vector:

$$d(k) = [\bar{q}_{d,1}(k), \ldots, \bar{q}_{d,n_d}(k), q_{dem,1}(k), \ldots, q_{dem,n_{in}}(k)]^T \in \mathbb{R}^{n_d+n_{in}} \tag{11}$$

where $\bar{q}_{d,j}$, j=1, ..., n_d denotes the nominal flow through uncontrolled gate j and q_{dem,n_i}, i=1, ..., n_{in} denotes the traffic demand emerging at controlled gate i.

The *input* vector is in the form:

$$u(k) = [q_{in,1}(k), \ldots, q_{in,n_{in}}(k)]^T \in \mathbb{R}^{n_{in}} \tag{12}$$

where $q_{in,j}$, j=1, ..., n_{in} denotes the gated flow through gate j.

2.4 Case study

Network description The proposed model is applied for a model network in Vissim. The model network is located in the 6th district of Budapest, and all its streets are one-way streets, and the average link length is 0.143 [km], thus the concept of the NFD can be applied for the network. The network can be entered via six controlled and six uncontrolled gates, and escaped via nine exit gates as depicted in Fig. 1.

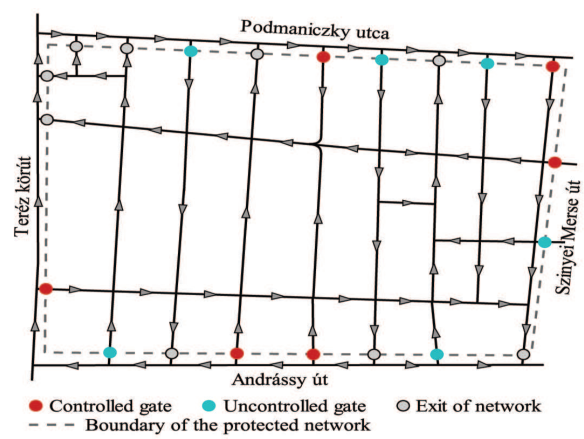

Figure 1: Network layout

Identification of network parameters Firstly, the network fundamental diagram (NFD) is identified for the model network. For this end, simulations were run with different traffic demands, representing the low demands and rush hours as well. The result of the eight different, one-hour-long simulation and the fitted fundamental function $F(\cdot)$ of Eq. (5) are plotted in Fig. 2.

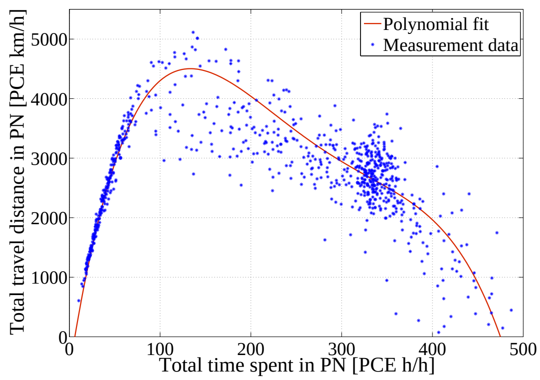

Figure 2: Fundamental diagram of the protected network

The best fit is obtained by a 4th-order polynomial function:

$$F(TTS) = -1.472{\cdot}10^{-8}\,TTS^4{+}1.657{\cdot}10^{-5}\,TTS^3 - 0.006568\,TTS^2{+}0.9659\,TTS{-}7.073 \qquad (13)$$

The second step is the identification of model parameter Γ based on the relationship in Eq. (6). This is carried out by a linear regression between the measurement data of Q_{out} and TTS_{PN}. Using that the average network link length is L=0.143 [km], the linear regression results in the following formula:

$$\Gamma{=}\begin{cases}0, & \text{if } TTD_{PN}{\le}45\,[PCEkm/h] \\ 0.06343{\cdot}TTD_{PN}{-}25.83, & \text{otherwise}\end{cases} \qquad (14)$$

The regression is illustrated by Fig. 3.

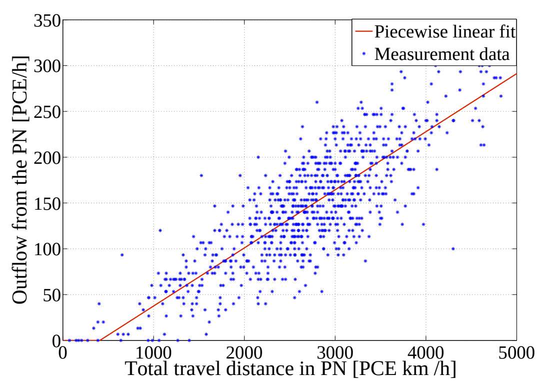

Figure 3: Linear regression for parameter Γ

The third parameter to be identified is β_j of Eq. (7). This parameter describes the relationship between the intended and realized inflow through a gate. In our approach, an average parameter β is considered representing β_j for each j gate. The measurement data and linear regression are plotted in Fig. 4. The result of the fitting is β=0.971.

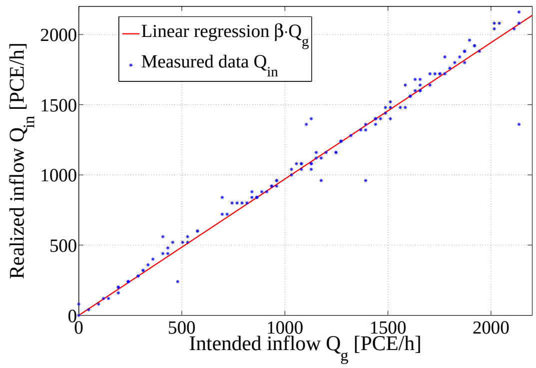

Figure 4: Linear regression for parameter β

3 Emission modeling in urban networks

The modeling of the vehicular emissions of a protected network follows the same approach as used in the traffic modeling framework: the overall emissions of the protected network is expressed using aggregated traffic variables: total travel distances (TTD) and total travel times (TTS).

The use of aggregated traffic variables for flow emission modeling has already been introduced in [Csikós et al. (2012)]. In that work, similarly to the traffic flow variables in the continuum models, emission of traffic is also considered as a function over space and time (i.e. as a distributed parameter system variable). Analogously to traffic performances (such as total travel distances (TTD) and total travel times (TTS), flow emission can be expressed as a function of macroscopic traffic variables in infinitesimal spatiotemporal rectangles using the following approach.

Consider a homogeneous traffic moving along a road and analyze the traffic variables on the short road segment $[l_0; l_0{+}L]$ for a short period of time $[t_0; t_0{+}T]$ (analysis in a spatiotemporal window of size $L{\times}T$). The traffic density in an infinitesimal spatiotemporal window $L{\times}T$ is equal to the Total Time Spent (TTS) in that window (for a detailed derivation see [Ashton (1966)]):

$$\rho_{[l_0;l_0+L]\times[t_0;t_0+T]} = \rho_{L\times T} = \frac{TTS_{L\times T}}{L{\cdot}T} \tag{15}$$

where $\rho_{L\times T}$ denotes the traffic density in $L{\times}T$. In a similar manner, average traffic flow in the spatiotemporal window is equal to the Total Travel Distance (TTD) in that window (for the derivation details, see [Ashton (1966)]):

$$q_{[l_0;l_0+L]\times[t_0;t_0+T]} = q_{L\times T} = \frac{TTD_{L\times T}}{L{\cdot}T} \tag{16}$$

where $q_{L\times T}$ denotes the traffic flow measured in $L{\times}T$.

This approach is extended to emission modeling in the following steps:

- The emission of a single vehicle is described by the emission factor function ef in unit $[g/km]$ (i.e. the distance specific emission) as a function of vehicle speed v. E.g. emission factor of pollutant CO can be considered with the following formula (using the model Copert IV [Ntziachristos et al.(2000)]):

$$ef_{CO} = \frac{\beta_m^{CO} v^m + \cdots + \beta_1^{CO} v + 1}{\alpha_n^{CO} v^m + \cdots + \alpha_1^{CO} v + 1} \tag{17}$$

  i.e. as a rational fractional function, in which parameters $\beta_m^{CO}, \ldots, \beta_1^{CO}$ and $\alpha_n^{CO}, \ldots, \alpha_1^{CO}$ are determined by curve fitting to vehicle dynamometer measurements of prespecified driving cycles.

- The overall emission of the traffic flow for pollutant p can be formalized within the spatiotemporal rectangle $L{\times}T$ in unit $[g]$ as follows:

$$E^p_{L\times T} = ef^p(v_{L\times T}){\cdot}TTD_{L\times T} \tag{18}$$

On a single road segment, substituting the generalized definition (16) to (18):

$$E^p_{L\times T} = ef^p(v_{L\times T}){\cdot}q_{L\times T}{\cdot}L{\cdot}T \tag{19}$$

Thus, the total produced emission of traffic can be expressed as a function of traffic flow and traffic mean speed for a spatiotemporal rectangle $L{\times}T$.

Remark: The model so far is simply applicable for lumped motorway systems, in a spatiotemporally discrete framework where the default unit is a motorway segment of length L during a sample time step of length T.

- In the following, the notation L_{PN} represents the links within the protected network:

$$L_{PN} = \{l_i\}, \quad i = 1, ..., n_{link} \tag{20}$$

The total emission in the protected network is the sum of the emissions of the network links:

$$E^p_{L_{PN}\times T} = \sum_{i=1}^{n_{link}} E^p_{l_i\times T} \tag{21}$$

However, the emission of the protected network PN needs to be calculated not by links, but for the whole network, using the aggregated variables TTS_{PN} and TTD_{PN}. Although the available measurements include the link-wise traffic flows, from which TTD_{PN} is calculated (see (3)), and the number of vehicles within the network is available from (4), average speeds are not supposed to be measured for each link i in our work. Nevertheless, average cruising speed in unit $[km/h]$ of the network can be expressed using the basic relationship among the traffic variables (see [Ashton (1966)]):

$$v_{L_{PN}\times T} = \frac{q_{L_{PN}\times T}}{\rho_{L_{PN}\times T}} \tag{22}$$

Substituting (15) and (16) to (22):

$$v_{L_{PN}\times T} = \frac{TTD_{L_{PN}\times T}}{TTS_{L_{PN}\times T}} \tag{23}$$

The average cruising speed v_{PN} is supposed to represent the speed conditions of the protected network in the spatiotemporal rectangle $L_{PN}{\times}T$, and is substituted to the framework.

- The emission of pollutant p, emerging in the protected network during time T can be stated as follows, using (23) and (18):

$$E^p_{L_{PN}\times T} = ef^p(v_{L_{PN}\times T}){\cdot}TTD_{L_{PN}\times T} \tag{24}$$

where $v_{L_{PN}\times T}$ is calculated as in (23). Emission of pollutant p in the protected network, in spatiotemporally discrete form:

$$E^p_{PN}(k) = ef^p(v_{PN}(k)){\cdot}TTD_{PN}(k) \tag{25}$$

where

$$v_{PN}(k) = \frac{TTD_{PN}(k)}{TTS_{PN}(k)} \tag{26}$$

By using the above formulae, the emission of the protected network can be stated using aggregated network parameters $v_{PN}(k)$ and $TTD_{PN}(k)$. However, it needs to be analyzed, how the extension of infinitesimal spatiotemporal increments L and T effects the accuracy of emission calculation. For this end, simulations are run in which the microscopic emission of the vehicles, and link-wise emissions of the traffic (21) are simulated. The calculated emissions (using eqs. (25)) are compared to the link-wise calculations and the reference, the microscopic emissions.

Remark: the emission of the vehicles stuck outside is considered with zero emissions. This consideration can be justified by the assumption of the presence of start-stop engine systems which is a wide spread accessory of modern vehicles. Moreover, this action can also be suggested by fixed message signs at the gates.

4 Analysis of model accuracy based on simulations

The accuracy of the developed model framework is analyzed through simulations. The suggested model framework, i.e. the modeling of the emission of the PN using aggregated traffic variables, is utilized with the Copert IV average speed model. This emission calculation is compared to two levels of emission modeling:

- The emission as a sum of each link's emission, calculated by macroscopic measurements of the links (link-wise emission modeling), also using the Copert IV average speed modeling for link emissions.
- The microscopic description which is considered reference, using the Versit+Micro model (see [Smit et al.(2007)]) via the EnViVer add-on module of Vissim.

Two scenarios are used for the comparison. The first scenario represents a rush hour situation with changing traffic loads and a fixed time signal control. The second scenario realizes the same traffic load, (i.e. a rush hour traffic with changing loads), but uses a PID controller with oscillations. The oscillations provide an opportunity to analyze the model accuracy for different state values.

Simulation environment

A microscopic traffic simulator: Vissim was used alongside its offline microscopic emission calculation add-on, EnViVer. EnViVer is based on the microscopic emission model Versit+Micro [Smit et al.(2007)]. Vissim enables signal controller commanding and loop-detector measurements through the COM interface by external programs like Matlab. The properties of the used framework are detailed in [Tettamanti ez al. (2012)].

For the simulations, the following parameters are set. The sampling time is chosen as the signal controller cycle time: $T_S = 90$s. The applied control parameters are detailed in the Appendix. The simulations run for 7200s. The emission factor function of the simulation is as follows (considering the following vehicle type: passenger car with an Euro 5 gasoline engine, under 1.4 l engine displacement):

$$ef(v) = \frac{\alpha + \gamma v + \epsilon v^2}{1 + \beta v + \delta v^2}; \tag{27}$$

with the following parameters: $\alpha = 0.5247$; $\beta = 0$; $\gamma = -0.01$; $\delta = 0$; $\epsilon = 9.36 \cdot 10^{-5}$.

Simulation results - case study no. 1

The accuracy of the model can be best examined, if a wide range of the state domain is used within a simulation. For this end, two scenarios are modeled. First, a congested situation is presented by scenario 1.

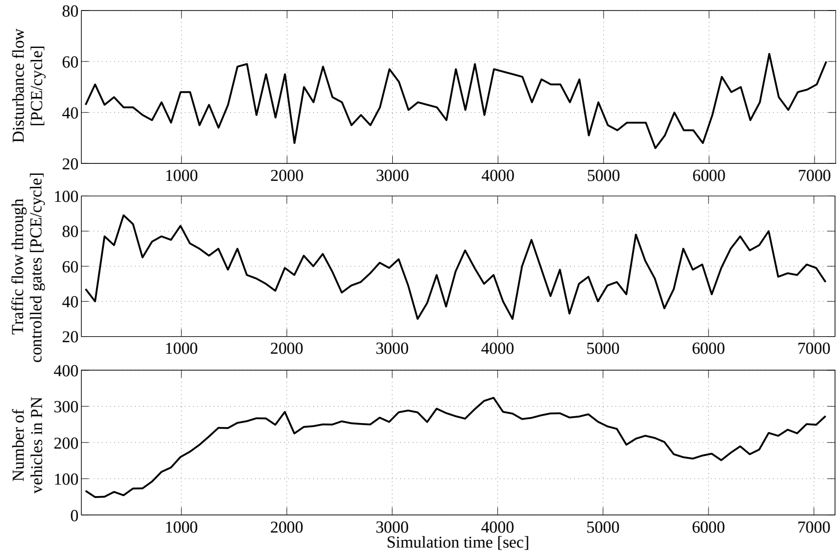

Figure 5: Case study 1: gate inputs and number of vehicles in PN

The gate control and disturbance signals of scenario no. 1 are plotted in Fig. 5, whereas the network performances (TTD, network average speed) are presented in Fig. 6.

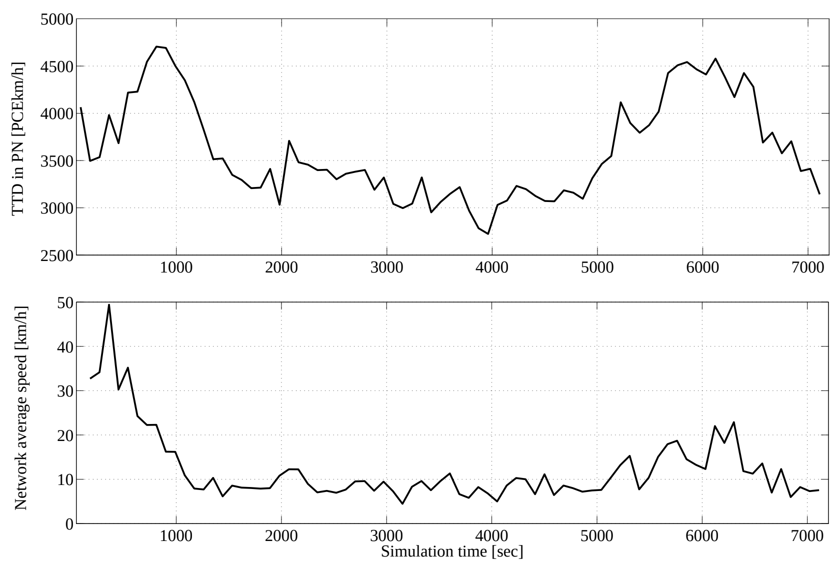

Figure 6: Case study 1: network performances

This basic scenario simulates congesting conditions, thus mainly low speeds are present. Fig. 7 highlights the model accuracy. The emission using aggregated variables (and network average speed) is very similar to the link-wise emission calculation. However, both has higher variations than the real emissions, calculated by Versit+Micro. The reason for this is the high sampling time (equal to the cycle time). By reducing the sampling time, the variation can be reduced. The relative error statistics of scenario no. 1 are summarized in table 1. The relative errors of the case study are high relative

	Link-wise emission	Emission based on aggregated variables
Average relative error	16.7 %	18.2 %

Table 1: Relative error results of case study no. 1 compared to the microscopic model

to the reference emission produced by the microscopic model. Nevertheless, the link-wise emissions and the emission values based on network-average traffic variables are very similar with small (<2 % relative errors). Thus the network average emission model shows good accuracy in scenario 1.

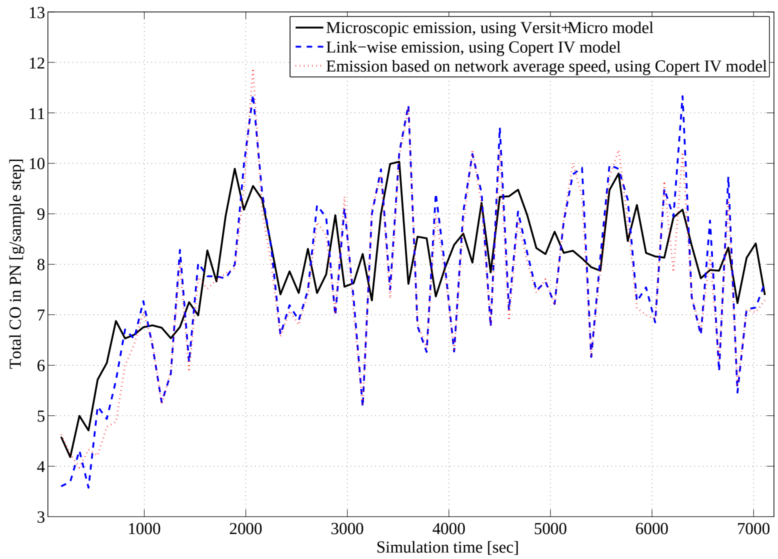

Figure 7: Case study 1: network emissions

Simulation results - case study no. 2

Scenario no. 2 features a PID controller, which is capable of preventing the congestion, however, with oscillations. Thus, a wide range of the domain is covered by the state dynamics and accuracy can be analyzed in case of extreme state values.

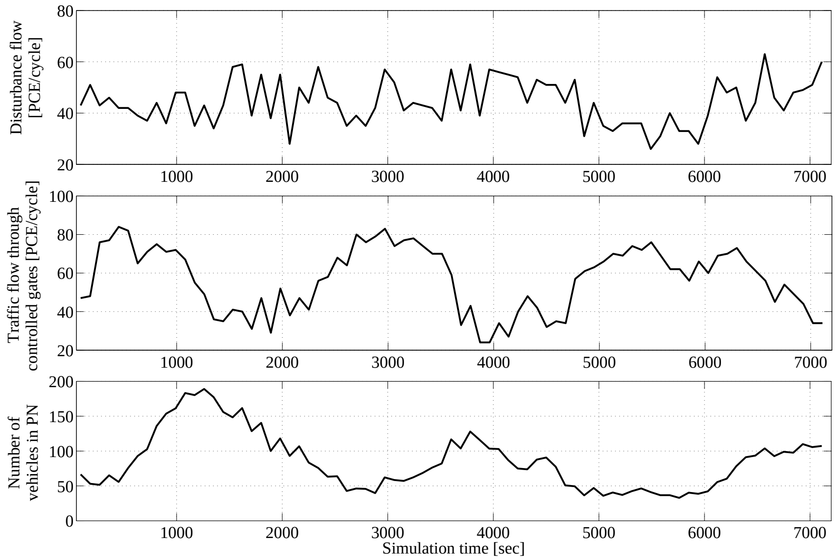

Figure 8: Case study 2: gate inputs and number of vehicles in PN

The gate control and disturbance signals of scenario no. 2 are plotted in Fig. 8, whereas the network performances (TTD, network average speed) are presented in Fig. 9.

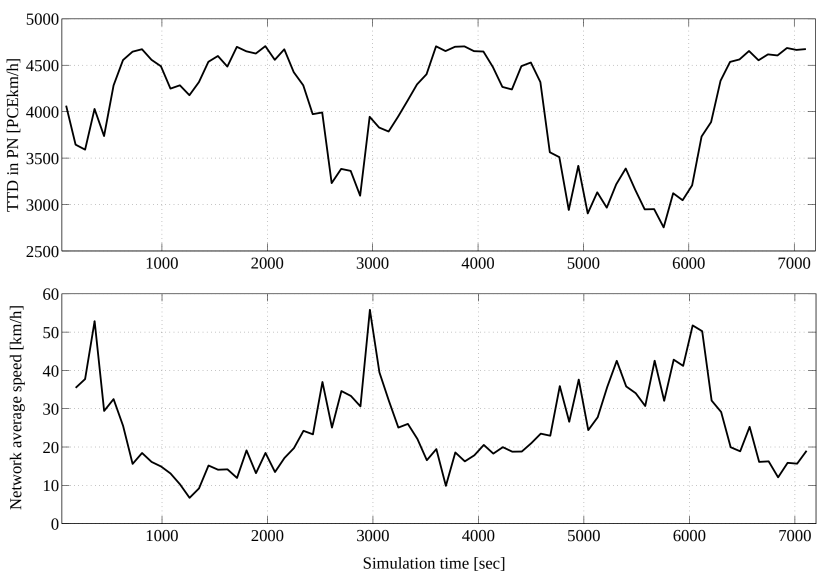

Figure 9: Case study 2: network performances

In this scenario both high and low traffic accumulation is present, thus the accuracy can be analyzed through both high and low speeds. Fig. 10 highlights the model accuracy. In this case, the emission using aggregated variables shows less resemblance to the link-wise emission calculation. Again, higher variations can be present in both macroscopic emissions relative to the real emissions, calculated by Versit+Micro. The relative error statistics of scenario no. 2 are summarized in table 2.

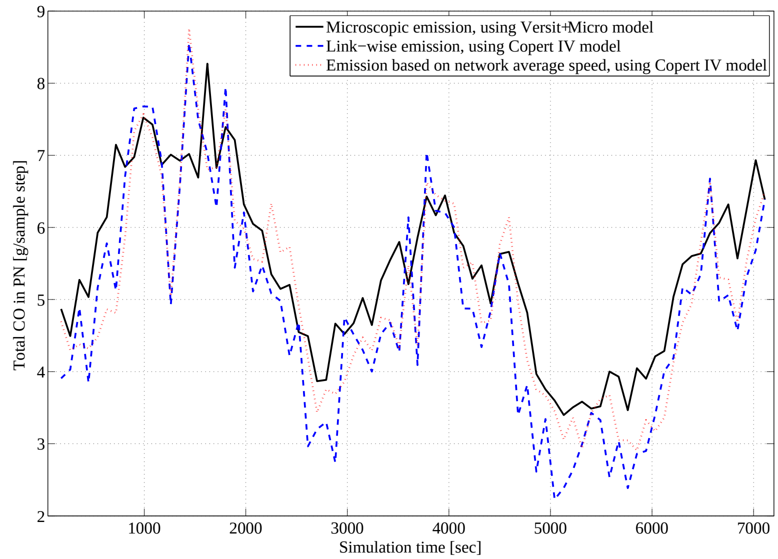

Figure 10: Case study 2: network emissions

	Link-wise emission	Emission based on aggregated variables
Average relative error	17.2 %	19.7 %

Table 2: Relative error results of case study no. 2 compared to the microscopic model

In scenario no 2. link-wise emissions and the emission values based on network-average traffic variables are very similar again, with small (<2 % relative errors). Thus the network average emission model shows good accuracy in scenario 2 as well.

5 Control objective statement

For a future multicriteria control, design is carried out for two criteria. The first control objective is the optimization of the traffic performance: maximizing the total travel distance within the PN, while minimizing the queues at perimeter gates. The second control objective is the minimization of traffic emissions within the PN. The objectives for the control criteria are stated separately, and turned to regulator-type control objective functions. By using the recast formulae, an overall cost function is composed as a weighted sum of the criteria. In this section the control objective statement for emission optimization within the PN is stated.

Emission optimization
The control goal is the minimization of network emissions in each step:

$$J_{em} = \sum_{k=1}^{K} \|E_{PN}(k)\|_2^2 = \sum_{k=1}^{K} \|ef(v_{PN}(k))TTD_{PN}(k)\|_2^2 \tag{28}$$

This cost function needs to be recast in the following form, to get a regulator problem:

$$J_{em} = \sum_{k=1}^{K} \|E_{PN}(N_{PN})(k)\|_2^2 \tag{29}$$

i.e. as a function of the state variable: the number of vehicles in PN (or equally, the TTS in the protected network). By using the model functions (5) and (2), the function of $TTD_{PN} = f(N_{PN})$ in eq. (28) is already stated. Furthermore, the composition of functions $ef(v_{PN}(N_{PN}))$ also needs to be formalized.

First, $ef(v)$ is analyzed. Eq. (17) presents the emission factor function, which is a monotonously decreasing function in the speed domain of urban driving cycles (see also [Gois et al. (2007)]). Fig. 11 illustrates the example of an emission factor function.

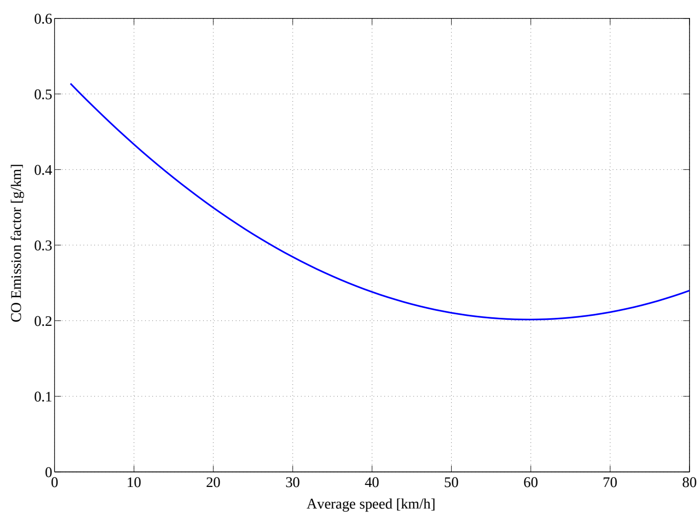

Figure 11: Emission factor function of CO pollution (passenger car equipped with an Euro 5 gasoline engine, under 1.4 l)

Second, $v(N_{PN})$ is analyzed. The relationship is often used in traffic modeling, as the equlibrium speed-density function of the first order macroscopic traffic description. In case of an urban network, the interpretation of the phenomenon 'equilibrium speed' is not straightforward. Nevertheless, analogously to the the NFD diagram, a relationship can be observed between the network average speed and the number of vehicles in the network.

For illustration, similarly to the NFD diagram, a fourth-order polynomial fitting was applied for the case study dataset. The network average speed is modeled as a function of TTS (or equally, the number of vehicles within the network), by the following formula:

$$v(TTS) = -3.681{\cdot}10^{-10}{\cdot}TTS^4 - 4.924{\cdot}10^{-8}{\cdot}TTS^3 + 0.0004515{\cdot}TTS^2 - 0.2671{\cdot}TTS + 49.95 \quad (30)$$

The measurement data and the polynomial fit of (30) is plotted in Fig. 12.

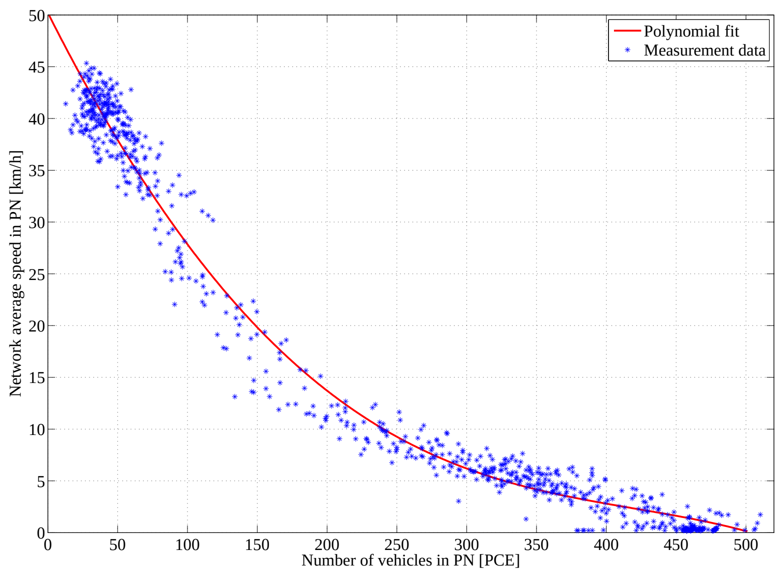

Figure 12: Network average speed function

Substituting the network average speed function(30) to eq. (4), the composition of functions $ef(v_{PN}(N_{PN}))$ can be stated in explicit form.

By using (30), (4) and (5), the objective function (28) can be formalized as a function of the state variable N_{PN}:

$$J_{em} = \sum_{k=1}^{K} \| ef(v_{PN}(N_{PN}(k))TTD_{PN}(N_{PN}(k)) \|_2^2 \quad (31)$$

The function stated in (31) alongside with emission data of simulations is plotted in Fig. 13. The function has a minimum at $N_{PN} = 0$, which is used as the regulator setpoint.

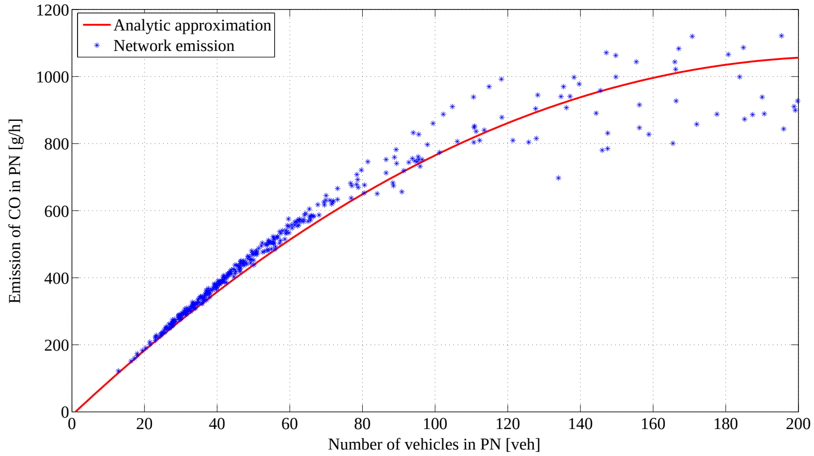

Figure 13: Network emission as a function of number of vehicles

By itself, cost function (13) is not applicable for a control system as the optimal performance could be reached by keeping the traffic outside the PN. However, in a multicriteria control design, it can be featured as a control criterion.

6 Conclusion

This work suggests a framework for modeling the traffic emissions in urban networks that are described by the network fundamental diagram (NFD) concept. The approach is the same as in [Csikós et al. (2012)]: traffic emission is formalized in finite spatiotemporal windows as a function of aggregated traffic variables (i.e. total travel distances in the network and network average speed). The framework is extended for the size of an urban network during a signal cycle - the size of a window in which the network aggregated parameters are modeled in the NFD concept. Simulations are carried out for model accuracy analysis, using the microscopic Versit+Micro model as reference. By using the emission model function and the traffic modeling relationships, the control objective for pollution reduction is formalized as a regulator problem for a model predictive control framework.

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