Stability Analysis and Control of Hybrid Systems

PhD thesis

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1 Motivation and results

Modelling of physical systems is of vital importance both in areas of theoretical research and in industrial applications. System models are usually developed either to understand (analyse) or control (synthesise). The ones used in industry are often based on, or originated from first engineering principles, see [1].

Models of physical systems can be described using various tools including linear or nonlinear approaches. The number of nonlinear representations is countless and currently it is practically hopeless to find universal analytical techniques which are applicable to any nonlinear system with arbitrary input signal. Among the existing tools, however, one of the most important type of description is the representation by ordinary differential equations.

Many applications demand, in addition, the ability of expressing a sort of discrete switching behaviour: while in nature nonlinearity is the rule rather than the exception, the same is true for being hybrid system among industrial applications. Hybrid systems are dynamic systems that exhibit both continuous and discrete dynamic behaviour, i.e., systems that can both flow (described by differential equations) and jump (described by difference equations or control graphs).

One approach is to describe a large class of hybrid systems is called *switched systems approach,* when some non-smooth variables are embedded into the continuous part (see [2]) in which the dynamics can be described by piecewise functions which are continuous, but not necessarily differentiable on the border between different dynamics.

Important problem is to check the stability of hybrid systems. There are procedures known both for discrete and continuous systems to decide whether or not a given hybrid system is stable. However it is not easily to find the domain of attraction (DOA) neither of nonlinear systems nor of hybrid systems in particular. In complicated cases often the only method to find the DOA is to examine point-wise a certain neighbourhood of the equilibrium point.

In this dissertation a method is presented to find estimation for the domain of attraction (DOA) of nonlinear autonomous systems. The presented method is based on by following the ideas of Vanelli and Vidyasagar in [3] that there exists a sequence of special kind of Lyapunov functions V_m which can be used to estimate the DOA. The elements of this sequence converge to a Lyapunov function of special kind and an iterative method is presented to find these V_m's. Also the DOA of the nuclear reactor in Paks Nuclear Power Plant is investigated in two different working modes. The considered subsystem is shown to be working within its DOA.

A method is presented based on the Vanelly-Vidyasagar-algorithm to find the DOA of a class of piecewise continuous systems. The DOA of a hybrid reactor model is also determined by pure analytic methods which utilize certain properties of the system form.

Another important task is to control hybrid systems. A classical approach for this is to solve the corresponding generalized quasi-variational inequalities (generalized forms of hybrid Bellman-equations) by adding costs accordingly to the jumps and changes of states. There has been notable progress in this area, but the methods to solve them still suffer from the curse of dimensionality.

In this dissertation a case study is given to demonstrate the process of designing a controller for a simple hybrid process system. The effect of several control parameters to be chosen is investigated on the controller performance and also the complexity and the performance of the controller are analysed. The chosen system for controller design is a hybrid model of the vaporizer in Paks Nuclear Power Plant. The investigated system belongs to a special subclass of hybrid systems. These systems have continuous states but their inputs are discrete, they are piecewise-constant due to technological reasons and have only *finite number of possible values.* It is, however, possible to design optimal control using MPT-techniques described (see[4]).

1.1 Notations

The notations and abbreviations used throughout the work are summarized in this section.

Notation	-	Meaning
A	-	matrix or set
A'	-	transpose of matrix A
$A \succeq 0$	-	matrix A is positive semi-definite
$A \succ 0$	-	matrix A is positive definite
I	-	identity matrix
c'	-	transpose of vector c
$u(t)$, or u	-	input of a given system
$y(t)$, or y	-	output of a given system
$\dot{x} = \frac{dx}{dt}$	-	time derivative of x
$\frac{\partial f(x)}{\partial x_i}$	-	i-th partial derivative of $f(x)$
∇x	-	gradient vector of x
$\mathscr{P}(A)$	-	power set of set A
$\mathscr{I}(A)$	-	interior of set A
$\bar{A}$	-	closure of set A
∂A	-	boundary of set A
$\|x\|_l$	-	l-norm of x, where l can be 1, 2 or ∞
$\mathbb{R}_0^+$	-	non-negative real numbers
$\mathbb{N}$	-	non-negative integers

The abbreviations used in the sequel are the followings.

Abbreviation		Meaning
PWA	-	piecewise affine
DOA	-	domain of attraction
MPT	-	multi-parametric programming technique
CFTOC	-	constrained finite time optimal control
RHC	-	receding horizon policy

2 Estimation of DOA of nonlinear autonomous systems

In the following we will consider the ordinary autonomous differential system in the form of

$$\dot{x} = f(x), \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is continuous. Moreover we assume that for each $x \in \mathbb{R}^n$ a unique solution $\phi(t, x)$ exists which is defined such that $\phi(0, x) = x$. Then the uniqueness of solutions implies that $\phi(t_1, \phi(t_2, x)) = \phi(t_1 + t_2, x)$ for $t_1, t_2 \in \mathbb{R}$ and considered as a function from $\mathbb{R} \times \mathbb{R}^n$ into $\mathbb{R}^n$, whereϕ is continuous in its arguments (see [5]). Note that the conditions on solutions $\phi(t, x)$ are true, for example, if function f is global of Lipschitz-type, i.e., there is a positive k such that $\|f(x) - f(y)\| \leq k \|x - y\|$ (see [6]).

The core concepts of the apparatus are given below.

Definition 1. The *domain of attraction* of the origin is the set

$$A = \{\, x_0 : x(t, x_0) \to 0 \text{ as } t \to \infty \,\}, \tag{2}$$

where $x(t, x_0)$ denotes the solution of the system in Eq. (1) corresponding to the initial condition $x(0) = x_0$.

Definition 2. A positive definite function V on an open neighbourhood U of the origin is said to be a *Lyapunov-function* for $\dot{x} = f(x)$ if $\dot{V}(x) = (\nabla V(x))' f(x) = \|f(x)\| \cdot \|\nabla f(x)\| \cos(\theta) \leq 0$ for all $x \in U \setminus \{0\}$, where θ is the angle between $f(x)$ and $\nabla V(x)$. When $\dot{V}(x) < 0$ for all $x \in U \setminus \{0\}$, the function V is called *strict Lyapunov-function*.

It is well known that even if a Lyapunov function exists to an autonomous ODE, then it is not unique. A maximal Lyapunov function is a special Lyapunov function on A which indicates the DOA for a given locally asymptotically stable origin.

Definition 3. A function $V_M : \mathbb{R}^n \to \mathbb{R}_0^+$ is called *maximal Lyapunov function* for the system (1) if

- $V_M(0) = 0$, $V_M(x) > 0$, $x \in A \backslash \{0\}$
- $V_M(x) < \infty$ if and only if $x \in A$
- $\dot{V}_M$ is negative definite over A and
- $V_M(x) \to \infty$ as $x \to \partial A$ and/or $\|x\| \to \infty$,

with A being the DOA of the origin for system (1).

The following theorem (see [3]) grounds the algorithm to find the DOA for a nonlinear autonomous system.

Theorem 4. *Suppose we can find a set $B \subseteq \mathbb{R}^n$ containing the origin in its interior and a continuous function $V : B \to \mathbb{R}_0^+$ and a positive definite function $\psi : \mathbb{R}^n \to \mathbb{R}_0^+$ such that*

- *$V(0) = 0$ and $V(x) > 0$ for all $x \in B \backslash \{0\}$,*
- *the function $\dot{V}(x_0) = \lim_{t \to 0^+} \frac{V(\phi(t,x_0)) - V(x_0)}{t}$ is well defined at all $x \in B$ and satisfies the relation $\dot{V}(x) = -\psi(x), \forall x \in B$ and*
- *$V(x) \to \infty$ as $x \to \partial B$ and/or $|\,\|x\| \to \infty$.*

Then $B = A$.

Suppose that function f can be expressed in Taylor series expansion

$$\dot{x} = f(x) = \sum_{i=1}^{\infty} F_i(x), \tag{3}$$

where functions F_i, $i \geq 1$ are homogeneous functions of degree i. For $i = 1$ it is $F_1(x) = \Phi x$, $\Phi \in \mathbb{R}^{n \times n}$, where Φ is the Jacobian matrix of f at $x = 0$. For the sake of brevity, let $F_i(x) = 0$, $i \leq 0$. This assumption makes it possible to express the forthcoming equations in simpler and more concise form, because at some combinations of the summation indices there appear F_is with non-positive i's.

Based on the properties of the maximal Lyapunov functions, we seek for a function V_M and a positive definite function ψ satisfying $V_M(0) = 0$ and

$$\dot{V}_M(x) = -\psi(x) \tag{4}$$

over some neighbourhood of the origin such that the set ∂A is given by the relation $V_M(x) \to \infty$.

The candidate Lyapunov function should excess any limit as x gets closer to the boundary of set A or as $\|x\| \to \infty$. If it was possible to write V_M as $V_M(x) = \frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomials in x then ∂A would be given by $D(x) = 0$. This construction leads to a structure which can in short be expressed as an under-determined set of linear equations

$$E_m y = b_m, \tag{5}$$

where E_ms are matrices of appropriate dimension, and the vector y is composed of the coefficients of the homogeneous functions R_is and Q_is.

Let us find homogeneous functions R_m and Q_{m-2}, $m \geq 3$, such that the coefficients of R_i and Q_i in the expression of $\dot{V}_m$ solve the following constrained minimization problem yielded by Eq. (5)

$$\begin{array}{c} \min e_m(y) \\ \text{s.t. } E_m(y) = b_m, \end{array} \tag{6}$$

where $e_m(y)$ is the squared 2-norm of the coefficients for the terms of degree greater than or equal to $m+1$ in the expression of $\dot{V}_m$.

According to the theorem of LaSalle [7] about invariant sets, one can choose the largest positive value $C^\star$ such that the sub-level set

$$A_{\text{est}} = \{ x : V_m(x) < C^\star \} \tag{7}$$

is contained in the region given by

$$\Omega = \left\{ x : \dot{V}_m(x) \le 0 \right\}. \tag{8}$$

As soon as the desired accuracy (computed as $e_m(y^\star) = 0$ for the minimizer $y^\star$) is reached (or the error starts growing) the iteration can be stopped.

Relation $A_{\text{est}} \subseteq A$ holds true in each step which means the more step one executes the more precise estimation of A can be computed.

If $e_m(y^\star) = 0$ for some $y^\star$ and some m then the iteration can be stopped and due to the relation $\dot{V}_m = -x'\mathfrak{Q}x$ the boundary of set A is given by $D(x) = 0$, i.e.,

$$A = \left\{ x : \sum_{i=1}^{m-2} Q_i(x) > -1 \right\}. \tag{9}$$

The advantage of this algorithm is that one does not have to know the solution of the system starting from different initial values; only a minimization problem (a linear programming problem) needs to be solved in each step of the iterative approximation procedure. Moreover, the applicable system class is wider than that of the majority of available algorithms can handle (they are mainly restricted to polynomial systems).

3 The DOA of hybrid nonlinear systems

Let us consider the class of piecewise defined hybrid systems in the following form:

$$\dot{x}_i = f_i(x), \tag{10}$$

where $f_i \subseteq \mathbb{R}^n \to \mathbb{R}^n$ represents the piecewise-defined system over finite number of different domains $X_i \subseteq \mathbb{R}^n$ with no point belonging to more than one dynamics, more precisely $f(x) = f_i(x)$, $x \in X_i \subseteq \operatorname{dom}(f_i)$, $i \in \bar{m} = \{1, 2, \ldots, m\}$ and $\cup_{i\in\bar{m}} X_i = X$ with $X_i \cap X_j = \emptyset$, $i \ne j \in \bar{m}$.

The border of a subset of the domains of dynamics (X_i, $i \in \bar{m}$) is given by $\mathcal{B}_I = \cup \left\{ \overline{X_i} \cap \overline{X_j}, i \ne j \in I \right\}$, with $\{ i_1, i_2, \ldots, i_l \} = I \in \mathscr{P}(\bar{m})$ being the indices of the domains. The border of all dynamics is then denoted by $\mathcal{B} = \mathcal{B}_{\bar{m}}$.

3.1 Extending Vanelli-Vidyasagar algorithm to piecewise hybrid systems

Let the DOA of the origin for the subsystem $\dot{x} = f_i(x)$ be denoted by A_i $(= A_i(\{0\}))$. Note that A_i is not narrowed onto X_i (on which the given sub-dynamics is active), i.e., $A_i \subseteq \operatorname{dom}(f_i)$.

To find the DOA $A = A(\{0\})$ for the system (10), the following criteria should be fulfilled for f:

1. f is continuous (can be piecewise-defined and may not be differentiable everywhere),

2. $f_i(0) = 0, \forall i \in \bar{m}$ (each subsystem has the origin as equilibrium point),

3. $0 \in \mathscr{I}(\cap_{i\in\bar{m}} A_i)$,

4. f should be Lipschitz-continuous, which due to Lagrange-theorem, holds if f_i is Lipschitz-continuous on X_i for each $i \in \bar{m}$ (and continuous on X according to point 1), and

5. in order to directly use the implemented algorithm in *Mathematica*, f should be differentiable on some neighbourhood about the origin.

It is seen from conditions (1)-(5) that if f is continuously differentiable about the origin or the origin lies within $\mathscr{I}\left(\cap_{i\in\bar{m}}X_i\right)$ then the corrected Vanelli-Vidyasagar-algorithm (and the corresponding implemented *Mathematica*-package) can be used on the hybrid system determined by f.

Based on the following theorem a computationally feasible algorithm can be given for finding a good approximation of the widest possible sub-level set for the DOA.

Proposition 5. *Let us consider the subsystem $\dot{x} = f_i(x)$ with its equilibrium point in $x = 0$. Let $\epsilon_i > 0$ such that the ball $S[\epsilon_i]$ is a compact subset of $\mathscr{I}(A_i)$ and define $h_i(\epsilon) = \min\{V_i(x) : x \in H(\epsilon)\}$ being the minimal value of the Lyapunov function on the surface of the ball. So we have $0 < h_i(\epsilon_i)$. If we choose α_i such that $0 < \alpha_i < h_i(\epsilon_i)$, then $P_{\alpha_i} = K_{\alpha_i} \cap S[\epsilon_i]$ is a compact positively invariant subset of A_i, where $K_{\alpha_i} = \{x \in A_i : V_i(x) \leq \alpha_i\}$ is the α_i sub-level set of the Lyapunov function $V_i(x)$.*

With the above proposition the way of finding the widest possible sub-level set is as follows.

Algorithm 6. *Let us select a hypersphere $H(r)$ about the origin which contains subset of the DOA A for the whole hybrid system. First we choose the maximal $r > 0$ such that the hypersphere of radius r lies inside $\mathcal{B}$ that is positively invariant. More precisely, we select the maximal $r > 0$ such that $(H(r) \cap \mathcal{B}) \subseteq (\cup_{i\in\bar{m}}A_i \cap \mathcal{B})$ holds true which is possible since X is locally compact. As a next step, we find the maximal value C_i of V_i on $A_i \cap X_i \cap S[r]$ (this C_i is the value of $C^\star$ in Eq. (7)) so we can construct the set $\cup_{i\in\bar{m}}\{x \in A_i \cap X_i : V_i(x) \leq C_i\}$ which is a subset of A.*

The method constructs an overall Lyapunov function from the individual Lyapunov functions of the sub-dynamics in a rational function form by approximating a maximal Lyapunov function V_i. In addition, a potentially conservative but computationally feasible method is given to estimate the overall DOA from the individual sub-Lyapunov functions using a maximal fitting hypersphere. An method based on the manual tuning of the level value has also been proposed for two-dimensional systems.

3.2 Determination of the DOA for the hybrid reactor model

Following an other approach, the DOA of an existing industrial plant of great importance, the primary circuit of a nuclear power plant in Paks is investigated.

Figure 1 shows the different operating units of the primary circuit and their connections which are taken into account in the simplified model. The controllers are denoted by double rectangles, their input and output signals are shown by dashed lines and the sensors that provide on-line measurements are also indicated in the figure by small full rectangles.

For convenience, unique identifiers are used for the operating units in the subscript of their related variables and parameters, as well as in all related modelling items, such as assumptions, that are listed below:

R reactor

PC primary circuit

PR pressurizer

SG steam generator

The overall modelling assumptions specify the considered operating units and their general properties which are:

- The set of operating units considered in the simple dynamic model includes the water in the tubes of the primary circuit (PC) and the pressurizer (PR).
- The dynamic model of the operating units is derived from simplified energy balances constructed for a single balance volume that corresponds to the individual unit.
- The only considered controller in the simplified model is the pressure controller. All the other controllers (including the level controller in the pressurizer, and the controller of the turbines, main circulating pumps and other compressors and valves in the system) are assumed to be ideal, that is, they keep their reference values ideally, without any dynamics or delays.

- The mass in the primary circuit (M_{PC}) is constant.
- The reactor is a system with concentrated parameters.
- The so-called one-group diffusion-theoretical approach was considered.
- Only the late neutron emitting nuclei group was taken into account.

3.2.1 Conservation balances

The first dynamics originates from the internal energy conservation balance, and it describes the temperature of the liquid in the primary circuit T_{PC} assuming that its overall mass M_{PC} is held constant:

$$\frac{dT_{PC}}{dt} = \frac{W_R - 6K_{SG}^T\left(T_{PC} - T_{SG}^*\right) - W_{PC}^{\text{loss}}}{M_{PC}c_{PC}^p}, \tag{11}$$

where T_{SG}^* is the nominal steam generator temperature (a constant), W_R is the power of the reactor, K_{SG}^T is the heat transfer coefficient between the primary circuit and the steam generator, W_{PC}^{loss} is the PC heat loss, M_{PC} is the PC water mass, c_{PC}^p is the PC specific heat of the liquid approximated as that of the water (on 282°C).

The second differential equation shows the dynamics of the temperature of the water in the pressurizer T_{PR} that originates from the internal energy balance of the liquid in the pressurizer:

$$\frac{dT_{PR}}{dt} = \frac{h\left(W_R\right) - W_{PR}^{\text{loss}} + W_{PR}^{\text{heat}} - c_{PR}^p m_{PR}\left(W_R\right)T_{PR}}{M_{PR}c_{PR}^p}, \tag{12}$$

where W_{PR}^{loss} is the PR heat loss, W_{PR}^{heat} is the heating power of the pressurizer, c_{PR}^p is the water specific heat (on 282°C) in the pressurizer and $h\left(W_R\right)$ describes the hybrid behaviour depending on the in/out mass flow from the primary circuit $m_{PR}\left(W_R\right)$ (see Eq. (17) later).

3.2.2 Algebraic constitutive equations

The reactor power W_R is proportional to the neutron flux N

$$W_R\left(v\right) = c_\psi N\left(v\right) \tag{13}$$

where c_ψ is a given constant.

The PR liquid mass M_{PR} is computed by assuming that the density of the liquid (ρ_{PC}) is a known quadratic function of the temperature, i.e.,and can

$$M_{PR} = M_{PC} - V_{PC}^0\left(c_{\varphi_0} + c_{\varphi_1}\tilde{T}_{PC} + c_{\varphi_2}\tilde{T}_{PC}{}^2\right), \tag{14}$$

where M_{PC} is the mass of the PC water, V_{PC}^0 is a nominal volume of the primary circuit, $\tilde{T}_{PC}$ is the PC temperature measured in Celsius, and c_{φ_0}, c_{φ_1} and c_{φ_2} are constants.

The PR heating power W_{PR}^{heat} is controlled by a proportional (P) controller as a function of the deviation of the temperature T_{PR} (that is proportional to the pressure) in the pressurizer from its reference value T_{PR}^*

$$W_{PR}^{\text{heat}} = W_{PR}^{\text{loss}} - K_{PR}\left(T_{PR} - T_{PR}^*\right), \tag{15}$$

where K_{PR} is a feedback gain of the pressure controller.

The in/out mass flow $m_{PR}\left(W_R\right)$ from the primary circuit is computed as the time-derivative of M_{PR}:

$$m_{PR}\left(W_R\right) = -V_{PC}^0\left(c_{\varphi_1} + 2T_{PC}c_{\varphi_2}\right)\frac{dT_{PC}}{dt}, \tag{16}$$

where the values of identified constants c_{φ_1} and c_{φ_2} (see also [8]).

The *hybrid behaviour* depends also on W_R:

$$h(W_R) = \begin{cases} c_{PC}^p m_{PR}\left(W_R\right)T_{PC}^{\text{hotleg}} & \text{for } m_{PR}\left(W_R\right) > 0 \\ c_{PR}^p m_{PR}\left(W_R\right)T_{PR} & \text{otherwise} \end{cases} \tag{17}$$

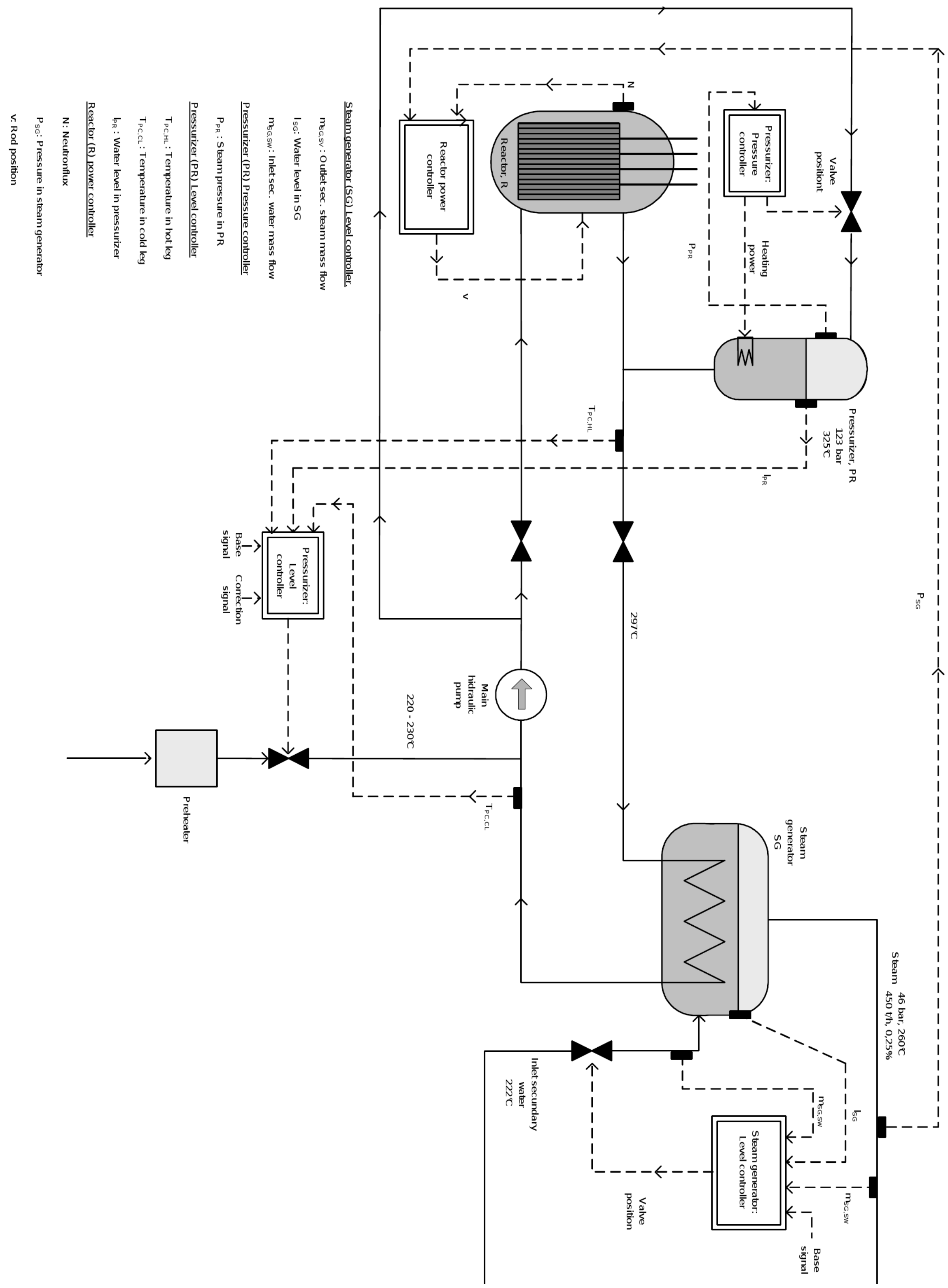


Figure 1: Process flow-sheet with the operating units of the simplified model

where $T_{PC}^{\text{hotleg}} = T_{PC} + 15$ is the temperature of the hot leg water.

The method used to determine the DOA is based on *purely analytical* considerations. A two dimensional autonomous hybrid differential equation model of the controlled primary circuit is developed for the purpose of the stability analysis.

It is shown that the DOA of the controlled system encapsulates the operating domain for any positive value of the considered feedback gain. It is also shown that the gain has a great impact on the dynamics of the controlled system and a practically advantageous domain of the gains has been determined.

4 Application of multi-parametric programming to controller design for discrete input hybrid nonlinear systems

Here another aspect of hybrid systems is considered: the task is not to find the DOA of a system (again, a certain kind of hybrid systems), but to design a controller for that.

The chosen system for controller design is a hybrid model of the vaporizer in Paks Nuclear Power Plant. A controller is designed which keeps the pressure in the tank on a constant level by manipulating the discrete input of the system.

The investigated system belongs to a special subclass of hybrid systems. These systems have continuous states but their inputs are discrete, they are piecewise-constant due to technological reasons and have only *finite number of possible values*. It is, however, possible to design optimal control using MPT-techniques described in [4] and implemented in [9].

Therefore, a case study is given to demonstrate the process of designing a controller for a simple hybrid process system. This approach (multi-parametric programming) in case of controlling system elements of a nuclear power plant was new at the time of this work [10].

4.1 System description

The simplified model consists of two energy balances: one for the water and and another one for the wall of the tank as balance volumes.

Water energy balance

$$\frac{dU}{dt} = c_p m T_I - c_p m T + K_W (T_W - T) + \sum_{i=1}^{4} \chi_i W_{HE_i}, \tag{18}$$

where U is the internal energy, c_p is the specific heat, m is the mass flow-rate of the water, T_I and T are the inlet and water temperatures, K_W is the heat transfer coefficient, T_W is the wall temperature, W_{HE_i} is the heat power and χ_i is the "on" indicator of the ith heating element.

Wall energy balance

$$\frac{dU_W}{dt} = K_W (T - T_W) - W_{loss}, \tag{19}$$

where U_W is the internal energy of the wall and W_{loss} is the overall energy loss in unit time.

The following *constitutive equations* describe the relationship between the internal energies (U and U_W) and the corresponding temperatures (T and T_W), as well as the dependence of the pressure p on the water temperature complete the model.

$$\begin{aligned} U &= c_p M T && (20)\\ U_W &= C_{pW} T_W && (21)\\ p &= p^*(T) = e^{p(T)}, && (22) \end{aligned}$$

where $p(T)$ is a given 3rd order polynomial, M is the mass of the water and $C_{pW} = c_{pW} M_W$ is the heat capacity of the wall:

$$p(T) = 6.5358 \times 10^{-1} + 4.8902 \times 10^{-2} T + 9.2658 \times 10^{-5} T^2 + 7.6835 \times 10^{-8} T^3.$$

The model parameters have been determined by using measured data from the industrial vaporizer system (see [11]) and can be found in table 1.

The state-space model may be regarded separately for each possible situation with the four heating elements being on or off, which means that the system is in hybrid state $\mathcal{H}_i$ when i number of heating elements are on. Thus the model above is a *piece-wise affine hybrid model*, where the discrete heating is responsible for the hybrid behaviour.

4.2 Controller design

The physical model and the controller is designed to keep its inner pressure (through the temperature of the tank T) within a narrow region by switching the heating elements on and off. A state-of-the-art Matlab-toolbox called MPT (Multi-Parametric Toolbox) was used to create the controller [9].

First, the continuous time model equations have been discretised with sampling time of τ seconds. The resulting discrete time PWA system is given in the form of

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \Phi \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \Gamma u(k) + \Theta \\ y(k) &= C \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \end{aligned} \tag{23}$$

where $\Phi = e^{A\tau}$, $\Gamma = \Xi B$, $\Theta = \Xi f$, where $\Xi = A^{-1}(\Phi - I)$ and τ is the sampling time.

The state-feedback controller designed for the discretised form comes as a set of polyhedra and a PWA control-law given (if no feedback pre-stabilization is enabled) in the next form:

$$U(k) = F_i^r x(k) + G_i^r, \tag{24}$$

where r denotes the active polytopic region from the set $\{\mathcal{R}_r\}_{r=1}^{Z}$ in which the controller actually is. The dynamics which is active in a particular partition is denoted by i which is constant 1 in the case of the investigated model of the pressurizer. The cost J associated to the state $x(k)$ is calculated by the formula

$$J = x(k)' \mathcal{A}_i^r x(k) + \mathcal{B}_i^r x(k) + \mathcal{C}_i^r, \tag{25}$$

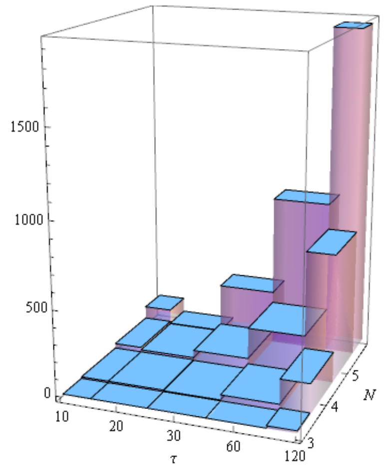
where matrices F_i^r, G_i^r, $\mathcal{A}_i^r$, $\mathcal{B}_i^r$, $\mathcal{C}_i^r$ and set of polytopes $\{\mathcal{R}_r\}_{r=1}^{Z}$ are yielded as the result of solving the CFTOC problem.

After solving the optimal control problem for a fixed prediction horizon N, evaluation of Eq. (24) gives a vector of control moves which minimizes the given performance criterion. When applying the obtained control law in the closed-loop system, only the first input $u(0)$ is extracted from the sequence and is applied on the system. This policy—of which procedure is iterated during the control process—is referenced to as the receding horizon policy (RHC).

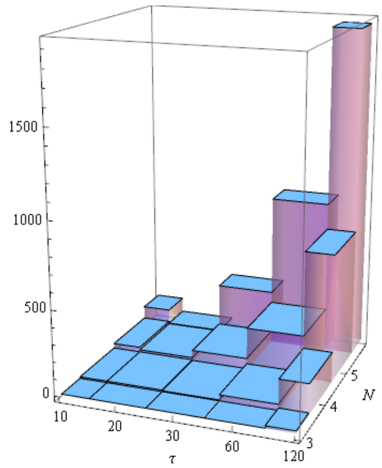
Norms For systems with discrete inputs solving the CFTOC problem with $l = 2$ is very slow. Moreover for the case of the vaporizer even when prediction horizon is 5 the output of the resulting closed-loop system does not converge to the reference point.

For the vaporizer system it is practically irrelevant what linear norm ($l = 1$ or $l = \infty$) is chosen. In effect the two controllers work identically. However infinity-norm has been chosen because the highest temperature matters the most between the water's and the wall's temperature.

Sampling time Choosing the value of sampling time has great effect on the controller. The smaller sampling time is chosen, the approximation of the continuous time model becomes more precise. On the other hand as the time-resolution increases, the complexity of the controller will increase as well. In addition, choosing small sampling times, the controller for the vaporizer cannot drive the system to the desired state. In the case of the investigated model it is good idea to choose $\tau = 60$ seconds which can be explained by the physics of the system. Due to the large mass hold-ups in the original system of industrial size, the time constants of the system are in the region of hours so sampling it with frequency of 1 minute is precise enough and still does not lead to too complex controller.



(a) Number of polytopes for 1-norm



(b) Number of polytopes for ∞-norm

Figure 2: Number of polytopes (Z) of the controller in function of sampling time (τ) and length of horizon (N)

If the sampling time is less than about $\tau = 20$ sec, the resulting controller cannot drive the system to the desired state thus it cannot track the reference output. In this case the controller does not work since the heating elements are all switched off and thus the system simmers down.

Using $\tau = 30$ sec as sampling time with prediction horizon $N = 5$ the controller works acceptable, however it has 321 polyhedral regions over the set of states that may be too complex solution in demanding environments.

Using $\tau = 60$ sec as sampling time with prediction horizon $N = 4$ leads to very good results. The number of regions in the controller is 258.

Using $\tau = 60$ sec as sampling time with prediction horizon $N = 3$ is enough to get good results. This final controller has only 41 regions that is significantly less than in the previous cases. That means choosing $\tau = 60$ seconds as sampling time with prediction horizon of $N = 3$ is a good choice from engineering viewpoint as well.

Length of prediction horizon The unit of the prediction horizon is 1 sampling instance and its length has great effect on the controller complexity: the longer the prediction horizon is, the more precise tracking the controller can obtain. The higher prediction horizon results in higher number of polytopic regions of the controller (see Fig. 2). A compromise has to be found between complexity and accuracy. The table below gives a short comparison on how the complexity increases as longer prediction horizon is chosen (for $l = \infty$):

N	2	3	4	5
#regions for $\tau = 30$	6	11	73	321
#regions for $\tau = 60$	6	41	258	902
#regions for $\tau = 120$	20	186	754	1924

To repeat the findings from the previous paragraph, prediction horizon $N = 3$ is a good choice for the compromise between complexity and accuracy.

notion	value	dimension	description
c_p	4183.232	J/kg/K	specific heat of water
ρ	654	kg/m^3	density of water on 325 Celsius
W_{HE_i}	90000	W	power of heating elements
M	17004	kg	mass of water in the tank
C_{pW}	2.5017×10^7	J/K	heat capacity of the wall
K_W	9.1894×10^4	W/K	heat transfer coefficient of the wall
T_I	267	°C	temperature of inlet water
W_{loss}	1.3231×10^5	W	heat loss

Table 1: Model parameters

5 New scientific results

The new scientific results presented are summarized in the following Theses.

Thesis 1. Estimation of DOA of nonlinear autonomous systems (Chapter 3) (**[R2]**,**[R3]**,**[R4]**)

An improved algorithm based on constructing maximal Lyapunov functions [3] is given to estimate the DOA of nonlinear autonomous systems. The advantage of this algorithm is that one does not have to know the solution of the system starting from different initial values; only a minimization problem (a linear programming problem) needs to be solved in each step of the iterative approximation procedure.

- The given algorithm can be applied on systems of finite dimension. However, due to the automatic calculation of $C^\star$ in Eq. (7), the DOA will reside inside an appropriately chosen sphere-environment of the origin, which can be considered to be a conservative approximation of the widest sub-level set.
- The proposed algorithm was used for an industrially relevant case-study. Two subsystems of the primary circuit in the Paks Nuclear Power Plant was investigated, one is the reactor characterized by neutron flux (N) and the other one is the steam generator. It was found that the DOA estimated by the proposed algorithm is almost exactly matches the real DOA.

Thesis 2. Estimation of DOA of hybrid nonlinear systems (Chapter 4) (**[R5]**,**[R6]**,**[R7]**)

Two different methods are proposed to estimate the DOA of two different types of switching hybrid systems that use the Lyapunov functions of the individual dynamics and construct the DOA therefrom.

- The first method is based on the algorithm proposed in Chapter 3, and it is capable to give a subset of the DOA for non-linear hybrid (switching) systems where the dynamics is continuous on the boundary of the different regimes of the state space. The method constructs an overall Lyapunov function from the individual Lyapunov functions of the sub-dynamics in a rational function form by approximating a maximal Lyapunov function. All necessary functions and algorithms have been implemented in a *Mathematica*-package.
- As an industrial relevant case study the DOA of an existing industrial plant of great importance, the primary circuit of a nuclear power plant was investigated. The method used to determine the DOA is based on purely analytical considerations. A two dimensional autonomous hybrid differential equation model of the controlled primary circuit was developed for the purpose of the stability analysis containing the temperatures of the liquid in the primary circuit and of the water in the pressurizer. It was shown that the DOA of the controlled system encapsulates the operating domain for any positive value of the considered feedback gain. It was also shown that the gain has a great

impact of the dynamics of the controlled system and a practically advantageous domain of the gains was determined.

Thesis 3. Application of multi-parametric programming to controller design for discrete input hybrid nonlinear systems (Chapter 5)
(**[R4]**)
In this thesis a PWA state-space model with discrete-valued inputs was developed for controller design purposes, where the number of possible input values is finite. The controller was designed by solving the corresponding CFTOC problem using multi-parametric programming. The effect of parameters of the model and also of the controller was investigated on the quality of the controller performance and computational properties.

- As an industrial relevant case study a simple two dimensional model was developed for the pressurizer of the nuclear reactor. The model has discrete inputs which makes it to fall to the hybrid model class. A controller for this model was designed by solving the CFTOC-problem using multi-parametric programming techniques.
- It can be ascertained that the prediction horizon and the sampling time are the two most important tuning parameters. To get shorter calculation times and to stabilize the closed-loop system it is highly encouraged to choose the longest sampling time and the shortest prediction horizon possible. The best controller-parameters were chosen by performing preliminary simulation experiments. It was found to be irrelevant whether 1-norm or infinity-norm is chosen.

6 Publications

The results of this thesis were previously either published in a journal or in form of research report or presented in conferences as enlisted below.

[R1] Sz. Rozgonyi, K. M. Hangos, Hybrid modelling and control of an industrial vaporizer, Proceedings of the 15th International Conference on Process Control, Slovakia, 2005., http://www.kirp.chtf.stuba.sk/pc05/data/index_papers.html#R

[R2] Sz. Rozgonyi, K. M. Hangos, Improved estimation method of region of stability for nonlinear autonomous systems, Proceedings of the 7th International PhD Workshop, Czech Republic, 2006., ISBN:80-903834-1-6, pp. 234-241

[R3] Sz. Rozgonyi, K. M. Hangos, G. Szederkényi, Estimating the stability region of a controlled pressurized water reactor, Proceedings of the 8th International Conference on The Modern Information Technology in the Innovation Processes of the Industrial Enterprises, Budapest, 2006., pp. 391-396

[R4] Sz. Rozgonyi, K. M. Hangos, G. Szederkényi, Improved estimation method of region of stability for nonlinear autonomous systems, Research Report, SCL-002/2006, Systems and Control Laboratory, Computer and Automation Research Institute, 2006., http://daedalus.scl.sztaki.hu/PCRG/PCRG_publist_ISO.html

[R5] Sz. Rozgonyi, K. M. Hangos, Estimating the region of stability for a hybrid model, Proceedings of the 16th International Conference on Process Control, Slovakia, 2007. On CD: 012s.pdf

[R6] Sz. Rozgonyi, K. M. Hangos, G. Szederkényi, Determining the domain of attraction of hybrid non-linear systems using maximal Lyapunov functions, Kybernetika 46 (1) (2010) 19–37, **IF:0.461**

[R7] Sz. Rozgonyi, K. M Hangos, Domain of attraction analysis of a controlled hybrid reactor model, Annals of Nuclear Energy 38 (5) (2011) 969–975, **IF:0.710**

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[10] S. Rozgonyi, K. M. Hangos, Hybrid modelling and control of an industrial vaporizer, in: Proceedings of the 15th International Conference on Process Control, Slovakia, 2005.

[11] K. Hangos, Z. Bordács, Modelling and identification of an industrial pressurized water tank (in hungarian), Tech. rep., Computer and Automation Research Institute, Budapest, Hungary (2004).